Prob. 1
Consider the following Cauchy-Euler D.E

\[ x^2y'' - xy' + 4y = 0 \]  

(a) Use the substitution \( x = e^t \) to transform (DE) to a differential equation with constant coefficients

(b) Solve the original equation (DE) by solving the new equation obtained in (a). Give the general solution as a function of the variable \( x \).
Prob. 2

Consider the following D.E:

\[(x^2 - 1)y'' - 2y = 0\] ((DE))

(a) Find a lower bound for the radius of convergence of power series solutions about the ordinary point \(x = 0\).

(b) Use power series to find a power series solution of (DE) about \(x = 0\).
Prob. 3
Consider the following differential equation

\[ 2xy'' - (1 + 2x^2)y' - xy = 0. \] ((DE))

(a) Is \( x = 0 \) a regular singular point? Justify your answer.
(b) Determine the indicial equation
(c) Find the indicial roots \( r_1 \) and \( r_2 \) corresponding to the singularity \( x = 0 \). Verify that \( r_1 \) and \( r_2 \) do not differ by an integer
(d) Use the method of Frobenius to find two linearly independent solutions \( y_1 \) and \( y_2 \) corresponding to \( r_1 \) and \( r_2 \), respectively.
Prob. 4
Find a general solution to the Cauchy-Euler equation

\[ x^2 y'' + 7xy' + 13y = 0. \]
Prob. 5
Solve

\[ y''' + 2y'' - 3y' = 0. \]
Prob. 6

Given the solution $y_1 = x$, find a general solution of

$$(x^2 - 1)y'' - 2xy' + 2y = 0, \ (x^2 < 1).$$

Do not use the formula. Show all the steps!
Prob. 7
Consider the problem

\[(D - 2)^3(D^2 + 9)y = x^2 e^{2x} + x \sin 3x.\]

(a) Find the complementary solution \(y_c\) (that is the general solution of the homogeneous equation)

(b) Find an annihilator (of the second member)

(c) Find the form of the particular solution (Do not compute the exact values of the constants, just determine the form)