Problem 1 (12 Points): Show that the equation is exact and solve. \( e^x \, dx + (x e^y + 2y) \, dy = 0 \).

Problem 2 (12 Points): Find the general solution of the Bernoulli equation
\[ x \, dy + y \, dx = xy^2 \, dx \, . \]
Problem 3 (12 Points): Solve the differential equation \( x \cos^2 y + \tan y \frac{dy}{dx} = 0 \).

Problem 4 (12 Points): Solve the differential equation \( \frac{dy}{dx} = \frac{2x + y + 1}{2x + y - 1} \).
Problem 5 (12 Points): For a substance C, the time rate of conversion is proportional to the square of the amount $x$ of unconverted substance present. If we start with 4 kg of the substance, and after one hour we found that three kgs. left, how long it will take to have only 1 kg left?

Problem 6 (12 Points): Consider the linear system of equations:

\[
\begin{align*}
3x + 8y - z &= -18 \\
2x + y + 5z &= 8 \\
2x + 4y + 2z &= -4
\end{align*}
\]

(a) Write the augmented matrix of the system.
(b) Use Gauss-Jordan elimination to write the matrix in reduced echelon form.
(c) Use part (b) to find the solution of the system.
**Problem 7 (10 Points):** Determine whether each of the following statements is true or false.

(a) For any matrix $A$, if $A^2 = 0$, then $A = 0$ (the zero matrix).
(b) For any matrix $A$, if $A^2 = I$, then $A = I$ (where $I$ is the identity matrix).
(c) $2(A + B) = 2B + 2A$ for any two matrices $A$ and $B$ with the same size.
(d) If $A^T = A^{-1}$ then $|A| = 1$ or $|A| = -1$.
(e) For any two matrices $A$ and $B$, if $AB = B$, then $A = I$ (the identity matrix).

**Problem 8 (18 Points):** Solve the system of equations

\[
\begin{align*}
2x + 2y &= 4 \\
2x - y &= 3
\end{align*}
\]

using

(a) the inverse of the coefficient matrix and

(b) Cramer’s Rule.