KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260 -FINAL EXAM

TIME: 12:30 - 3:30 P.M. Tuesday – September 1, 2009

Serial Number:________________ Section Number:_______________

Name:_____________________________________ Student Number:_______________

Important Notes

1. **Check that the exam paper has 12 different questions on 8 different pages.**
2. **Do not use calculators.**
3. **Show all your work. No credits for answers not supported by work.**

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1. (8 Points) Identify each of the following statements as true or false.
   (a) Any algebraic system of 5 equations and 5 unknowns has a unique solution.
   (b) Similar matrices have the same determinant.
   (c) The number of elements in a minimal spanning set of $\mathbb{R}^5$ is equal to the maximum number of linearly independent vectors.
   (d) Every matrix has a Jordan normal form.
   (e) The matrix A and its transpose A^T have the same eigenvalues.
   (f) The geometric multiplicity is equal to the algebraic multiplicity of any eigenvalue λ.
   (g) The matrix A has a zero eigenvalue iff $|A| = 0$.
   (h) If λ is an eigenvalue of the matrix A then λ^3 is an eigenvalue of A^3.

2. (8 Points) The student population $U = U(t)$ a school has time rate of change $\frac{dU}{dt}$, which is proportional to the difference between maximum population of 5,000 and U. If there are 2,500 students this year (t = 0) and 3,000 next year, what will be the population of the school in two years from now?

3. (8 Points) Find the general solution of the differential equation: $x\sin^2 y + e^x \frac{dy}{dx} = 0.$

4. (8 Points) Solve the initial value problem: $t^3 y' + 4t^2 y = e^{-t}, \quad y(-1) = 0$

5. (8 Points) Find the function $k(x)$ satisfying $k(0) = 0$ that makes the following equation exact differential equation: $[x^2 y + y\tan x]dx + [k(x) + x^2]dy = 0.$
   <u>**(DO NOT SOLVE THE EQUATION)**</u>

6. (8 Points) Use **either** the method of undetermined coefficients **or** variation of parameters to find the general solution of the differential equation: $y'' - 2y' + y = 4xe^x$.

7. (8 Points) Suppose that ***x***, ***y***, and ***z*** are three linearly independent vectors in $\mathbb{R}^3$. Determine whether the vectors { **2*x* – *y* , 2 *y* - *z* , 2*z* - *x*** } are linearly dependent or independent. Show your work.

8. (8 Points)Consider the system of equations $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -3 \\ 2 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ s \\ t \end{bmatrix}$.

   a. For what values of *s* and *t* does the system has a unique solution?
   b. Use Cramer's Rule to find the value of y when $s = t = 2$.

9. (8 Points) Consider the equation $y'' - y' - 6y = 0$.

   a. Write the equation as a linear system of first order.
   b. Check that the two vectors $X_1(t) = \begin{bmatrix} e^{3t} \\ 3e^{3t} \end{bmatrix}$ and $X_2(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$ generate the general solution of the system on $(-\infty, \infty)$.

10. (8 Points) Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

    a. Find the determinant of *A*.
    b. Find the eigenvalues and eigenvectors of *A*.
    c. Find an invertible matrix *P* and a diagonal matrix *D* such that $P^{-1}AP = D$. (**DO NOT FIND** P^{-1})
    d. Use Hamilton-Cayley theorem to find A^{-1}.

11. (10 Points) Find the **general solution** of the system

$$x' = 2x + y$$
$$y' = -x + 2y$$

Also find the solution to the initial value problem where $x(0) = 1$ and $y(0) = 2$

12. (10 Points) Find the **general solution** of the system $X' = AX$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$