

Show all your work. No credits for answers without work.

Problem 1: Determine if the following set of vectors is linearly dependent or independent in \mathbb{R}^3 .

$S_1 = \{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}$. Can you write the vector $\vec{v} = (3, 3, 3)$ as a linear combination of these vectors?

If yes show how.

Problem 2: Determine if the following sets form subspaces of \mathbb{R}^2 . Show why.

(a) $V_1 = \{(x, x^2) : x \in \mathbb{R}\}$

(b) $V_2 = \{(2x, -x) : x \in \mathbb{R}\}$

Problem 3: Identify each of the following statements as true or false.

- (a) Any set of more than 5 vectors is linearly independent in \mathbb{R}^5 .
- (b) Any set of 5 linearly independent vectors in \mathbb{R}^5 form a basis.
- (c) A minimal spanning set of vectors form a basis.
- (d) Any set containing the zero vector is linearly dependent.
- (e) If the system $A\vec{X} = \vec{0}$ has only one solution then the dimension of the solution space is 1.
- (f) A set of vectors is linearly independent if one of them can be written as a linear combination of the other vectors.
- (g) Any two spanning sets of a vector space must have the same number of elements.

Problem 4: find a basis for the solution space of the system $A\vec{X} = \vec{0}$, where

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 3 & -1 & 2 & 3 \\ 1 & 3 & -4 & 1 \end{bmatrix}$$