KFUPM | Term 091 | Date: 31/10/2009
Dept. of Math. & Stat. | MATH 101 | Duration: 15 minutes

Quiz 2

Name: ID #: Section 3 Serial #:

Q1. (4 points) Given that $F(r)=\begin{cases}\dfrac{GMr}{R^3} & \text{, if } r<R\\ \dfrac{GM}{r^2} & \text{, if } r\geq R\end{cases}$, $R>0$, is F continuous on $(-\infty,+\infty)$?

Check ONLY at $r=R$.

$\lim_{r\to R^-} F(r) = \lim_{r\to R^-}\frac{GMr}{R^3} = \frac{GM\cancel{R}}{R^{\cancel{3}}} = \frac{GM}{R^2}$ 1

$\lim_{r\to R^+} F(r) = \lim_{r\to R^+}\frac{GM}{r^2} = \frac{GM}{R^2}$ 1 $\Rightarrow$ 1 1

Since $\lim_{r\to R^-} F(r) = \lim_{r\to R^+} F(r) \Rightarrow F$ is cont. at $r=R \Rightarrow F$ is cont.

Q2. (4 points) Find $\lim_{x\to\infty} f(x)$ if $\frac{10e^x-21}{2e^x}<f(x)<\frac{5\sqrt{x}}{\sqrt{x-1}}$ $\forall x>1$.

Use squeezing theorem:

$\lim_{x\to\infty}\frac{10e^x-21}{2e^x} = \lim_{x\to\infty}\frac{10-\frac{21}{e^x}}{2}$ 1 $= \frac{10-\frac{21}{\lim_{x\to\infty} e^x}}{2} = \frac{10-\frac{21}{\infty}}{2} = 5$ 1

$\lim_{x\to\infty}\frac{5\sqrt{x}}{\sqrt{x-1}} = \lim_{x\to\infty}\frac{5}{\sqrt{\frac{x-1}{x}}}$ 1 $= \lim_{x\to\infty}\frac{5}{\sqrt{1-\frac{1}{x}}} = \frac{5}{\sqrt{1-\lim_{x\to\infty}\frac{1}{x}}} = \frac{5}{\sqrt{1-0}} = 5$ 1

$\Rightarrow \lim_{x\to\infty} f(x) = 0$ by Squeezing theorem

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Q3. (2 points) If $f'(a)=\lim_{u\to -8}\frac{u-u^{1/3}+6}{u+8}$, then find $f(u)$ and a.

Since $u\to -8 \Rightarrow a=-8$ 1

Let 1 $f(u)=u-u^{1/3} \Rightarrow f(-8) = -8-(-2) = -6 = f(a)$

$f'(-8) = \lim_{u\to -8}\frac{u-u^{1/3}-(-6)}{u-(-8)} = \lim_{u\to -8}\frac{u-u^{1/3}+6}{u+8}$

With My Best Wishes

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Quiz 2

Name: ID #: Section 49 Serial #:

Q1. (4 points) Show that the equation $\frac{1}{x^2-3x+2}=\frac{1}{3-2x-x^2}$ has at least one real solution.

Let $f(x) = \frac{1}{x^2-3x+2} - \frac{1}{3-2x-x^2} = \frac{1}{(x-2)(x-1)} + \frac{1}{(x+3)(x-1)}$

1 $= \frac{x+3+x-2}{(x-1)(x-2)(x+3)} = \frac{2x+1}{(x-1)(x-2)(x+3)}$

1 $f(0) = \frac{1}{(-1)(-2)(3)} = \frac{1}{6} > 0$, $f(-1) = \frac{-1}{(-2)(-3)(2)} = -\frac{1}{12} < 0$

$\Rightarrow$ 1 since 1. f is cont. on $[-1,0]$, 2. $f(-1)<0$ & $f(0)>0$ $\Rightarrow$

by IVT $\exists\, c\in(-1,0)$ s.t. $f(c)=0 \Rightarrow \frac{1}{c^2-3c+2} = \frac{1}{3-2c-c^2}$

1

Q2. (4 points) Find $\lim\limits_{x\to-\infty}\frac{\sqrt{4x^2+1}}{x+1}$.

$\lim\limits_{x\to-\infty}\frac{\sqrt{4x^2+1}}{x+1} = \lim\limits_{x\to-\infty}\frac{|x|\sqrt{4+\frac{1}{x^2}}}{x(1+\frac{1}{x})}$ 1 $=$

1 $= \lim\limits_{x\to-\infty}\frac{-x\sqrt{4+\frac{1}{x^2}}}{x(1+\frac{1}{x})} = \lim\limits_{x\to-\infty} -\frac{\sqrt{4+1/x^2}}{1+1/x}$ 1

$= -\frac{\sqrt{4+0}}{1+0} = \boxed{-2}$ 1

Q3. (2 points) If $f'(a)=\frac{21}{2}\lim\limits_{x\to\frac{5}{7}}\frac{\ln x-\ln 5+\ln 7}{7x-5}$, then find $f(x)$ and a.

since $x\to\frac{5}{7} \Rightarrow a=\frac{5}{7}$ 1 & $f'(a) = \lim\limits_{x\to\frac{5}{7}} \frac{21}{2}\frac{\ln x-(\ln 5-\ln 7)}{7(x-\frac{5}{7})}$

$\Rightarrow f'(a) = \lim\limits_{x\to\frac{5}{7}}\frac{\frac{3}{2}\ln x - \frac{3}{2}\ln(\frac{5}{7})}{x-\frac{5}{7}} \Rightarrow f(x) = \ln x^{3/2}$, $a=\frac{5}{7}$ 1

With My Best Wishes