KFUPM
Dept. of Math. & Stat.

Term 091
MATH 101
Quiz 5

Date: 13/1/2010
Duration: 25 minutes

Name: ID #: Section 3 Serial #:

Q1. (5 points) Find the relative extrema (if any!) of $f(x) = x^4(x-1)^3$.

$f'(x) = 4x^3(x-1)^3 + 3x^4(x-1)^2 = x^3(x-1)^2[4(x-1)+3x]$

$= x^3(x-1)^2(7x-4) = 0$ (1) at $x = \{0, 1, \frac{4}{7}\}$ (1)

x^3	$-$	$+$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$	$+$
$(7x-4)$	$-$	$-$	$+$	$+$
$f'(x)$	$+$	$-$	$+$	$+$

(1)

0 max, $\frac{4}{7}$ min, 1

So using FDT,

$\Rightarrow \boxed{x = 0}$ is a rel. max. & $\boxed{x = \frac{4}{7}}$ is a rel. min.

(1) (1)

Q2. (5 points) If $y = \ln\left(\frac{x}{4}\right)$ and x decreases from 4 to 3.9, then approximate the corresponding change in y.

(1) $\Delta y \approx dy = f'(x_0)\,\Delta x$, where $x_0 = 4$ (1) & $\Delta x = 3.9 - 4 = -0.1$ (1)

(1) $f'(x) = \frac{1}{4} \cdot \frac{4}{x} = \frac{1}{x}$

(1)

$\Rightarrow \Delta y \approx \frac{1}{4} \cdot (-0.1) = \boxed{-\frac{1}{40}} = -0.025$

Q3. (5 points) Find the points at which the tangent line to $y = \cosh(2x)$ has a slope of 2.

$y' = 2\sinh(2x) = 2 \Rightarrow \sinh(2x) = 1$ (1) $\Rightarrow$

$\frac{e^{2x} - e^{-2x}}{2} = 1 \Rightarrow$ (1) $e^{2x} - e^{-2x} = 2 \Rightarrow (e^{2x})^2 - 1 = 2e^{2x}$ (1), let $u = e^{2x}$

$\Rightarrow u^2 - 2u - 1 = 0 \Rightarrow u = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$ (1)

but $u > 0 \Rightarrow u = 1+\sqrt{2} \Rightarrow e^{2x} = 1+\sqrt{2} \Rightarrow 2x = \ln(1+\sqrt{2})$

$\Rightarrow \boxed{x = \frac{1}{2}\ln(1+\sqrt{2})} = \ln\sqrt{1+\sqrt{2}}$

(1) *With My Best Wishes*

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Name: ID #: Section 49 Serial #:

Q1. (5 points) Find the local extrema (if any!) for $g(t)=t^2(2t-5)^{1/3}$.

$$g'(t) = 2t(2t-5)^{1/3} + \frac{2}{3}t^2(2t-5)^{-2/3} = 2t(2t-5)^{1/3} + \frac{2t^2}{3(2t-5)^{2/3}}$$

$$= \frac{6t(2t-5)+2t^2}{(2t-5)^{2/3}} = \frac{14t^2-30t}{(2t-5)^{2/3}} = \frac{2t(7t-15)}{(2t-5)^{2/3}} \quad (1)$$

(1) $g'(t)$ DNE at $\boxed{t=\frac{5}{2}}$ & $g'(t)=0$ at $\boxed{t=0}$, $\boxed{t=\frac{15}{7}}$ are C.N (1)

$g'(t)$: $+$ (max) $-$ (min) $+$ $+$ at 0, $\frac{15}{7}$, $\frac{5}{2}$ $\Rightarrow$ $x=0$ is a rel. max (1), $x=\frac{15}{7}$ is a rel. min (1) by FDT.

Q2. (5 points) If $f(x)=\ln(x^2-1)$ then find the number(s) c in $(-\sqrt{2},\sqrt{2})$ that satisfies the Mean Value Theorem.

The Mean Value Theorem is NOT (1) satisfied because $f(x)$ (1) is (1) discont. on $[-\sqrt{2},\sqrt{2}]$ since $f'(x)$ exists on $(-\infty,-1)\cup(1,+\infty)$ so, such a c is not guaranteed, however, $f'(x)=\frac{2x}{x^2-1}$

$$f'(c)=\frac{f(\sqrt{2})-f(-\sqrt{2})}{\sqrt{2}+\sqrt{2}} \Rightarrow \frac{2c}{c^2-1}=\frac{0-0}{2\sqrt{2}}=0 \Rightarrow (1)\ \boxed{c=0} \in (-\sqrt{2},\sqrt{2}) \text{ BUT } 0 \notin dom(f). \ (1)$$

Q3. (5 points) If $f(x)=\operatorname{sech}^2(\ln(x+2))$, then find $f'(0)$.

$$f(x)=\frac{1}{\cosh^2(\ln(x+2))} = \frac{4}{\left[e^{\ln(x+2)}+e^{-\ln(x+2)}\right]^2} = \frac{4}{\left[x+2+(x+2)^{-1}\right]^2} \quad (1)$$

$$= \frac{4}{\left[x+2+\frac{1}{x+2}\right]^2} = \frac{4}{\left[\frac{(x+2)^2+1}{(x+2)^2}\right]^2} = \frac{4(x+2)^2}{(x^2+4x+5)^2} \quad (1)$$

$$\Rightarrow f'(x) = \frac{8(x+2)(x^2+4x+5)^2 - 4(x+2)^2\,2(2x+4)(x^2+4x+5)}{(x^2+4x+5)^4} \quad (1)$$

$$= \frac{8(x+2)(x^2+4x+5) - 8(2x+4)(x+2)^2}{(x^2+4x+5)^3} \ (1) = \frac{8(x+2)\left[x^2+4x+5-2x^2-8x\ldots\right]}{(x^2+4x+5)^3}$$

$$= \frac{-8(x+2)(x^2+4x+3)}{(x^2+4x+5)^3}$$

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$$\Rightarrow f'(0) = \frac{-8(2)(3)}{(5)^3} = \boxed{\frac{-48}{125}} \quad (1)$$