KFUPM SEM I (Term 091) Name: _____ Serial #: _____

MATH 101-4-6-17 Quiz # 4 ID: #: KEY Sec. #: _____

DO NOT SIMPLIFY the answer in Problems 1 and 2.

1. (3-points) Find $\frac{d}{dx}\left(\tan(3\sin(4\cos 2x)\right)$.

$$= \sec^2(3\sin(4\cos 2x)) \cdot 3\cos(4\cos 2x) \cdot [-8\sin 2x]$$

$$= -24\sec^2(3\sin(4\cos 2x)) \cdot \cos(4\cos 2x) \cdot (\sin 2x)$$

2. (3-points) $\frac{d}{dx}\sqrt{5+4x^2}\cos^{-1}\left(\frac{5}{x}\right)$.

$$= \sqrt{5+4x^2}\left(\frac{-\left(\frac{-5}{x^2}\right)}{\sqrt{1-\frac{25}{x^2}}}\right) + \frac{8x}{2\sqrt{5+4x^2}}\cos^{-1}\left(\frac{5}{x}\right).$$

$$= \frac{5\sqrt{5+4x^2}}{\sqrt{x^4-25x^2}} + \frac{4x\cos^{-1}\left(\frac{5}{x}\right)}{\sqrt{5+4x^2}}.$$

• 3. (4-points) Find the slope of the tangent line to the graph of $2xy + 3\sqrt{x^2 + y^2} = 39$ at the point $(3, 4)$.

We use implicit differentiation:

$$2xy' + 2y + \frac{3(2x+2yy')}{2\sqrt{x^2+y^2}} = 0$$

Let $x = 3$ and $y = 4 \Rightarrow$

$$6y' + 8y + \frac{3(3+4y')}{5} = 0 \Rightarrow$$

$$30y' + 40 + 9 + 12y' = 0 \Rightarrow$$

$$42y' = -49 \Rightarrow$$

$$y'|_{(3,5)} = -\frac{7}{6}.$$

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DO NOT SIMPLIFY the answer in Problems 1 and 2.

1. (3-points) Find $\frac{d}{dx}\left(\sin(2\tan(3\cos 4x)\right)$.

$= \cos(2\tan(3\cos 4x)) \cdot 2\sec^2(3\cos 4x) \cdot [-12\sin 4x]$

$= -24\cos(2\tan(3\cos 4x))\sec^2(3\cos 4x)\sin 4x$.

2. (3-points) $\frac{d}{dx}\sqrt{4+5x^2}\cos^{-1}\left(\frac{4}{x}\right)$.

$= \sqrt{4+5x^2} \cdot \frac{-\left(-\frac{4}{x^2}\right)}{\sqrt{1-\frac{16}{x^2}}} + \frac{10x}{2\sqrt{4+5x^2}}\cos^{-1}\left(\frac{4}{x}\right)$

$= \frac{4\sqrt{4+5x^2}}{\sqrt{x^4-16x^2}} + \frac{5x\cos^{-1}\left(\frac{4}{x}\right)}{\sqrt{4+5x^2}}$.

3. (4-points) Find the slope of the tangent line to the graph of $3xy + 2\sqrt{x^2 + y^2} = 46$ at the point $(3, 4)$.

We use implicit differentiation:

$$3xy' + 3y + 2\frac{(2x + 2yy')}{2\sqrt{x^2+y^2}} = 0$$

Let $x = 3$ and $y = 4 \Rightarrow$

$$9y' + 12 + \frac{6 + 8y'}{5} = 0$$

$$\Rightarrow 45y' + 60 + 6 + 8y' = 0$$

$$\Rightarrow 53y' = -66$$

$$\Rightarrow y'\Big|_{(3,4)} = -\frac{66}{53}$$ which is the required slope.

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DO NOT SIMPLIFY the answer in Problems 1 and 2.

1. (3-points) Find $\frac{d}{dx}\left(\cos(2\sin(3\tan 4x)\right)$.

$$= -\sin(2\sin(3\tan 4x)) \cdot 2\cos(3\tan 4x) \cdot 12\sec^2 4x$$

$$= -24\sin(2\sin(3\tan 4x)) \cdot \cos(3\tan 4x) \cdot \sec^2 4x.$$

2. (3-points) $\frac{d}{dx}\sqrt{2+3x^2}\sin^{-1}\left(\frac{2}{x}\right)$.

$$= \sqrt{2+3x^2} \cdot \frac{\left(-\frac{2}{x^2}\right)}{\sqrt{1-\frac{4}{x^2}}} + \frac{6x}{2\sqrt{2+3x^2}} \cdot \sin^{-1}\left(\frac{2}{x}\right)$$

$$= \frac{-2\sqrt{2+3x^2}}{\sqrt{x^4-4x^2}} + \frac{3x\sin^{-1}\left(\frac{2}{x}\right)}{\sqrt{2+3x^2}}.$$

•• 3. (4-points) Find the slope of the tangent line to the graph of $5xy+2\sqrt{x^2+y^2}=70$ at the point $(3,4)$.

We use implicit differentiation:

$$5xy' + 5y + 2\,\frac{2x+2yy'}{2\sqrt{x^2+y^2}} = 0$$

Let $x=3$ and $y=4$ $\Rightarrow$

$$15y' + 20 + \frac{6+2y'}{5} = 0 \Rightarrow$$

$$75y' + 100 + 6 + 2y' = 0 \Rightarrow$$

$$77y' = -106 \Rightarrow$$

$$y'\Big|_{(3,4)} = -\frac{106}{77}.$$