King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 101-**Quiz** 1 **Term** 091

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Exercise 1. Let a be a positive real number ($a > 0$). Find the limit of the function

$$f(t) = \frac{\sqrt{t^2 + a^2} - a}{t^2},$$

as t approaches 0.

Exercise 2. Find the vertical asymptotes of the curve $y = f(x)$, where

$$f(x) = \frac{1}{x^2 - 4x + 3}.$$

Solutions

Exercise 1.

The domain of the function f is $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$. Let us find $\lim_{t\to 0} f(t)$. Note that, we cannot apply the quotient law, since $\lim_{t\to 0} t^2 = 0$.

We have to simplify the quotient

$$\begin{aligned} f(t) &= \frac{\sqrt{t^2+a^2}-a}{t^2} \\ &= \frac{(\sqrt{t^2+a^2}-a)(\sqrt{t^2+a^2}+a)}{t^2(\sqrt{t^2+a^2}+a)} \\ &= \frac{(\sqrt{t^2+a^2})^2-a^2}{t^2(\sqrt{t^2+a^2}+a)} \\ &= \frac{t^2+a^2-a^2}{t^2(\sqrt{t^2+a^2}+a)} \\ &= \frac{t^2}{t^2(\sqrt{t^2+a^2}+a)} \\ &= \frac{1}{\sqrt{t^2+a^2}+a} \end{aligned}$$

This yields $\lim_{t\to 0} f(t) = \lim_{t\to 0} \frac{1}{\sqrt{t^2+a^2}+a} = \frac{1}{2a}$.

Exercise 2.

Clearly, we have $x^2 - 4x + 3 = (x-1)(x-3)$. Hence the domain of f is $\mathbb{R} \setminus \{1, 3\}$. Thus the potential vertical asymptotes of the curve $y = f(x)$ are $x = 1$ and $x = 3$.

As $x \to 1^-$, $\frac{1}{x-1} \to -\infty$ and $\frac{1}{x-3} \to -\frac{1}{4}$. It follows that $\lim_{x\to 1^-} f(x) = \infty$.

Also, if x is close to 1 but but larger than 1, $\frac{1}{x-1}$ is a large positive number and $\frac{1}{x-3}$ is close to $-\frac{1}{4}$. Therefore, $\lim_{x\to 1^+} f(x) = -\infty$.

Likewise, one may show that $\lim_{x\to 3^+} f(x) = \infty$ and $\lim_{x\to 3^-} f(x) = -\infty$.

We conclude that $x = 1$ and $x = 3$ are the vertical asymptotes of the curve $y = f(x)$.

Remak that, it is sufficient to check that

$(\lim_{x\to 1^-} f(x) = \infty$ or $\lim_{x\to 1^+} f(x) = -\infty)$ and $(\lim_{x\to 3^+} f(x) = \infty$ or $\lim_{x\to 3^-} f(x) = -\infty)$.