King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 101-**Quiz** 3 **Term** 091

Instructor: Prof. Othman Echi

NAME:.............................. ID:............... Section: 08

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Exercise. Let $f(x) = \sqrt[3]{x}$.

(1) If $a \neq 0$, find $f'(a)$.
(2) Show that f is not differentiable at 0.
(3) Show that $y = \sqrt[3]{x}$ has a vertical tangent line at $(0, 0)$.

Solutions

(1) Finding the derivative of f using the definition.

For each real numbers $x \neq a$, one may write

$$\begin{aligned}\frac{f(x)-f(a)}{x-a} &= \frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}\\ &= \frac{\sqrt[3]{x}-\sqrt[3]{a}}{(\sqrt[3]{x})^3-(\sqrt[3]{a})^3}\\ &= \frac{1}{(\sqrt[3]{x})^2+(\sqrt[3]{x})(\sqrt[3]{a})+(\sqrt[3]{a})^2}\ .\end{aligned}$$

Hence, for $a \neq 0$, we have

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{x\to a}\frac{1}{(\sqrt[3]{x})^2+(\sqrt[3]{x})(\sqrt[3]{a})+(\sqrt[3]{a})^2} = \frac{1}{3(\sqrt[3]{a})^2},$$

showing that f is differentiable at a and $f'(a) = \dfrac{1}{3(\sqrt[3]{a})^2} = \dfrac{1}{3}a^{-\frac{2}{3}}$.

(2) Differentiability at 0.

Consider the quotient $\dfrac{f(0+h)-f(0)}{h} = \dfrac{\sqrt[3]{h}}{h} = \dfrac{1}{h^{\frac{2}{3}}}$. Its limit as h approaches 0^+ (reps., 0^-) is $+\infty$ (reps., $-\infty$). Thus, f is not differentiable at 0.

(3) The curve $y = \sqrt[3]{x}$ has a vertical tangent line at $(0,0)$.

Clearly, the function f is continuous at 0. On the other hand,

$$\lim_{x\to 0}|f'(x)| = \lim_{x\to 0}\frac{1}{3}|x|^{-\frac{2}{3}} = +\infty.$$

This proves that f has a vertical tangent line at $(0,0)$.