King Fahd University of Petroleum and Minerals
Department of Math & Stat
Math 102 Section # 4, 5, 8 (091)
Quiz 3(a)

Time: 20 minutes Marks: ________/9

Name: "Solution" Section #: ________

ID #: ________ Serial #: ________

1. Use partial fractions to evaluate $\int \frac{x^3+6x^2+3x+6}{x^3+4x}\,dx$

$$\int \left[1 + \frac{6x^2 - x + 16}{x^3+4x}\right]dx \quad \text{[by long division]}$$

$$x^3+4x = x(x^2+4)$$

$$\int \left[1 + \frac{A}{x} + \frac{Bx+C}{x^2+4}\right]dx$$

$$\int \left[1 + \frac{4}{x} + \frac{2x-1}{x^2+4}\right] \quad \text{[by P.F.]}$$

$$x + 4\ln|x| + \ln(x^2+4) - \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

2. Evaluate $\int \sin x \sin 2x \sin 3x\, dx$.

$$\frac{1}{2}\int \sin x\left(\frac{1}{2}\left[\cos(2x-3x) - \cos(2x+3x)\right]\right)dx$$

$$\frac{1}{2}\int \frac{2}{2}\sin x \cos x\, dx - \frac{1}{2}\int \sin x \cos 5x\, dx$$

$$\frac{1}{4}\int \sin 2x\, dx - \frac{1}{2}\int \frac{1}{2}\left[\sin(x+5x) + \sin(x-5x)\right]dx$$

$$\frac{1}{4}\cdot\frac{1}{2}(-\cos 2x) - \frac{1}{4}\int(\sin 6x - \sin 4x)\,dx$$

$$-\frac{1}{8}\cos 2x - \frac{1}{4}\frac{1}{6}(-\cos 6x) + \frac{1}{4}\cdot\frac{1}{4}(-\cos 4x) + c$$

$$-\frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x + C$$

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1. Evaluate $I=\int \frac{2x^2+3}{x(x-1)^2}\,dx$.

$$\frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx \quad \text{—— } (*)$$

$x=0$ in $(*) \Rightarrow 3 = A$

$x=1$ in $(*) \Rightarrow 5 = C$

Comparing coefficients of powers of x in $(*) \Rightarrow$ $\left.\begin{array}{l} 2=A+B \\ 0=-2A-B+C \\ 3=A \end{array}\right] \Rightarrow C=5$, $B=-1$

$$I = \int \frac{3}{x}dx - \int \frac{dx}{x-1} + \int \frac{5}{(x-1)^2}dx$$

$$= 3\ln x - \ln(x-1) - 5/(x-1) + C$$

2. Determine whether $I=\int_{-\infty}^{\infty} \frac{x\,dx}{\sqrt{x^2+2}}$ converges or diverges. If converges, then find its sum.

$$I = \int_{-\infty}^{0} \frac{x}{\sqrt{x^2+2}}dx + \int_{0}^{\infty} \frac{x\,dx}{\sqrt{x^2+2}} = I_1 + I_2$$

$$I_2 = \lim_{t\to\infty} \int_0^t x(x^2+2)^{-1/2}dx$$

$$= \lim_{t\to\infty} \left[(x^2+2)^{1/2}\right]_0^t = \sqrt{\infty} - \sqrt{2} = \infty$$

Similarly $I_1 = -\infty$

$\Rightarrow I = I_1 + I_2 = -\infty + \infty$ (undefined).

$\Rightarrow I$ is divergent

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1. Evaluate $I = \int \frac{1}{x+\sqrt[3]{x}}\,dx$.

put $u = x^{1/3} \Rightarrow u^3 = x$ and $u^2 = x^{2/3}$

$du = \frac{1}{3x^{2/3}}\,dx \Rightarrow \frac{1}{3u^2}\,dx \Rightarrow 3u^2\,du = dx$.

$$I = \int \frac{3u^2\,du}{u^3+u} = \frac{3}{2}\int \frac{2u\,du}{u^2+1}$$

$$= \frac{3}{2}\ln(u^2+1) + C$$

$$= \frac{3}{2}\ln(x^{2/3}+1) + C.$$

2. Check whether $\int_3^\infty \frac{8}{x^2-4}\,dx$ converges or divergers. If converges, then find its sum.

$$\int_3^\infty \left[\frac{2}{x-2} - \frac{2}{x+2}\right]dx \qquad \text{(By P.F.)}$$

$$2\left[\lim_{t\to\infty}\int_3^t \left[\frac{1}{x-2} - \frac{1}{x+2}\right]dx\right.$$

$$2\lim_{t\to\infty}\left\{\Big[\ln(x-2) - \ln(x+2)\Big]_3^t\right\}$$

$$2\lim_{t\to\infty}\left[\ln\left(\frac{t-2}{t+2}\right) - 0 + \ln 5\right]$$

$$2\left[0 + \ln 5\right] = 2\ln 5$$

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1. Evaluate $I = \int \frac{dx}{1 - \sin x + \cos x}$.

put $u = \tan\frac{x}{2} \Rightarrow dx = \frac{2}{1+u^2}du$

$\sin x = \frac{2u}{1+u^2}$, $\cos x = \frac{1-u^2}{1+u^2}$

$$I = \int \frac{1}{1 - \frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}} \left(\frac{2}{1+u^2}\right) du = \int \frac{(1+u^2)}{\cancel{2}(1-u)} \frac{\cancel{2}}{(1+u^2)} du$$

$$= \int \frac{du}{1-u} = -\ln|1-u| + C = -\ln\left(1 - \tan\frac{x}{2}\right) + C.$$

2. Check whether $\int_{-\infty}^{0} \frac{x\,dx}{1+x^2}$ is convergent or divergent. If converges, then find its sum.

$$\lim_{t \to -\infty} \left[\frac{1}{2}\ln(1+x^2)\right]_t^0$$

$$\lim_{t \to -\infty} \left[\frac{1}{2}\ln(1) - \frac{1}{2}\ln(1+t^2)\right]$$

$$\left[0 - \frac{1}{2}\lim_{t \to -\infty}\ln(1+t^2)\right]$$

$$-\frac{1}{2}\ln\left(1+(-\infty)^2\right)$$

$$-\frac{1}{2}\ln(\infty) = -\infty$$

So the improper integral is divergent