King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 091
Saturday 30/1/2010
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\displaystyle\int \frac{(2-\sqrt{x})^2}{x}\,dx =$

(a) $x - 8\sqrt{x} + 4\ln|x| + C$

(b) $x - 2\sqrt{x} + 4\ln|x| + C$

(c) $x - \dfrac{8}{\sqrt{x}} + 4\ln|x| + C$

(d) $1 - 4\sqrt{x} + \ln|x| + C$

(e) $2x - \dfrac{1}{4}\sqrt{x} + 2\ln|x| + C$

2. $\displaystyle\int \frac{1}{x\sqrt{25-(\ln x)^2}}\,dx =$

(a) $\sin^{-1}\left(\dfrac{\ln x}{5}\right) + C$

(b) $\sin^{-1}\left(\dfrac{\ln x}{\sqrt{5}}\right) + C$

(c) $\dfrac{1}{5}\sin^{-1}(\ln x) + C$

(d) $\sin^{-1}\left(\cos\left(\dfrac{x}{5}\right)\right) + C$

(e) $\dfrac{1}{5}\sin^{-1}\left(\dfrac{\ln x}{5}\right) + C$

3. If $F(x) = \int_{\sqrt{x}}^{\sqrt{2}} \frac{2}{1+t^4}\,dt$, then $F'(x) =$

(a) $\frac{-1}{(1+x^2)\sqrt{x}}$

(b) $\frac{2}{5} - \frac{2}{1+x^2}$

(c) $\frac{2}{1+x^2}$

(d) $\frac{2}{1+x^4}$

(e) $\frac{2}{(1+x^2)\sqrt{x}}$

4. $\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot e^{-\frac{2i}{n}} =$

(a) $\int_0^2 \frac{1}{2} e^{-x} dx$

(b) $\int_0^2 e^{-2x} dx$

(c) $\int_0^1 2e^{-x} dx$

(d) $\int_0^1 e^{-\frac{x}{2}} dx$

(e) $\int_1^2 2e^{-x} dx$

5. The sum of the series $\sum_{n=0}^{+\infty} \frac{1}{3^n \cdot n!}$ is equal to

(a) $\sqrt[3]{e}$

(b) e^3

(c) $\sin(3)$

(d) $\sqrt[3]{e} - 1$

(e) e^{-3}

6. $\int_0^{\pi/4} \tan^4 x \, dx =$

(a) $\frac{\pi}{4} - \frac{2}{3}$

(b) $\frac{\pi}{3} - \frac{1}{2}$

(c) $\frac{\pi}{2} + \frac{1}{2}$

(d) $\frac{\pi}{4} - \frac{1}{2}$

(e) $\frac{\pi}{3} + \frac{2}{3}$

7. The coefficient of x^{10} in the Maclaurin series of $f(x) = \sin(x^2)$ is equal to

(a) $\frac{1}{120}$

(b) 0

(c) $\frac{-1}{6}$

(d) $\frac{1}{6}$

(e) $\frac{1}{10}$

8. The improper integral $\int_{-\infty}^{1} \frac{x}{(1+x^2)^3}\,dx$ is

(a) convergent and its value is $\frac{-1}{16}$

(b) convergent and its value is $\frac{2}{9}$

(c) convergent and its value is $\frac{-3}{8}$

(d) convergent and its value is $\frac{3}{16}$

(e) divergent

9. If the line $x = a$ divides the region bounded by the curves

$$y = \frac{1}{x^2}\ ,\ y = 0, x = 1,\ \text{and}\ x = 4$$

into two regions with equal area, then $a =$

(a) $\frac{8}{5}$

(b) 2

(c) $\frac{8}{3}$

(d) $\frac{4}{3}$

(e) 3

10. The interval of convergence I and the radius of convergence R of the power series

$$\sum_{n=1}^{+\infty} \frac{(x-1)^n}{n \cdot 2^n}$$

are

(a) $I = [-1, 3)$ and $R = 2$

(b) $I = (-1, 3)$ and $R = 2$

(c) $I = (-1, 1)$ and $R = 1$

(d) $I = [-2, 2]$ and $R = 2$

(e) $I = [-1, 3]$ and $R = 2$

11. The volume of the solid obtained by rotating the region enclosed by the curves

$$y = e^x, y = 0, x = 0, \text{ and } x = 1$$

about the y-axis is equal to

(a) 2π

(b) π

(c) 3π

(d) $\frac{\pi}{2}$

(e) $\frac{3\pi}{2}$

12. If $\dfrac{8x^2 + 7x - 6}{x^2(x+3)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3}$, then $A + B + C =$

(a) 6

(b) -5

(c) 3

(d) 0

(e) -1

13. The series $\sum_{n=1}^{+\infty} e^{-n} \cdot n!$

(a) diverges by the ratio test

(b) converges by the ratio test

(c) diverges by the integral test

(d) converges by the comparison test

(e) converges by the test for divergence

14. The first three terms of the Taylor series of $f(x) = \cos(2x)$ about $a = \pi$ are given by

(a) $1 - 2(x-\pi)^2 + \frac{2}{3}(x-\pi)^4$

(b) $1 - 2(x-\pi) - 2(x-\pi)^2$

(c) $1 - 2(x-\pi)^2 + 16(x-\pi)^4$

(d) $-1 + 2(x-\pi) + \frac{4}{3}(x-\pi)^3$

(e) $1 + 2(x+\pi)^2 - \frac{2}{3}(x+\pi)^4$

15. The sequence $\left\{ \dfrac{(2n-1)! \cdot (5n^2+2n)}{(2n+1)!} \right\}_{n=1}^{+\infty}$

(a) converges and its limit is $\dfrac{5}{4}$

(b) converges and its limit is 5

(c) converges and its limit is 1

(d) converges and its limit is $\dfrac{5}{2}$

(e) is divergent

16. The series $\sum_{n=1}^{+\infty} (-1)^n \dfrac{n+27}{n+28}$ is

(a) divergent

(b) absolutely convergent

(c) conditionally convergent

(d) divergent by the integral test

(e) convergent by the ratio test

17. The smallest number of terms of the series $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(2n+1)^4}$ that we need to add so that $|\text{error}| < 0.0001$ is

(a) 4

(b) 40

(c) 400

(d) 10

(e) 22

18. $\int_2^4 \frac{\sqrt{x^2-4}}{x}\, dx =$

(a) $2\sqrt{3} - \frac{2\pi}{3}$

(b) $1 - \frac{2\pi}{3}$

(c) $\sqrt{3} - \pi$

(d) $2\sqrt{3} + \frac{\pi}{3}$

(e) $\sqrt{3} - \frac{\pi}{2}$

19. Which one of the following statements is **TRUE**:

(a) If $a_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{3}$, then $\sum_{n=1}^{+\infty} a_n$ is convergent

(b) The series $\sum_{n=1}^{+\infty} n^{-\pi}$ is divergent

(c) If $0 < a_n \leq b_n$ for all n and $\sum_{n=1}^{+\infty} b_n$ diverges, then $\sum_{n=1}^{+\infty} a_n$ diverges

(d) If $\lim_{n\to+\infty} a_n = 0$, then $\sum_{n=1}^{+\infty} a_n$ is convergent

(e) If $\lim_{n\to+\infty} \sqrt[n]{|a_n|} = 1$, then $\sum_{n=1}^{+\infty} a_n$ is divergent

20. $\int_0^1 |4x - 3| \, dx =$

(a) $\frac{5}{4}$

(b) $\frac{7}{8}$

(c) $\frac{3}{8}$

(d) 2

(e) $\frac{3}{4}$

21. The series $\sum_{n=1}^{+\infty} \frac{n^{2n}}{(1+2n^2)^n}$

(a) converges by the root test

(b) diverges by the root test

(c) is a series with which the root test is inconclusive

(d) diverges by the test of divergence

(e) diverges by the comparison test

22. $\int \frac{\sqrt{x}}{x+4}\, dx =$

(a) $2\sqrt{x} - 4\tan^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$

(b) $2\sqrt{x} + 2\tan^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$

(c) $\sqrt{x} - 2\tan^{-1}(\sqrt{x}) + C$

(d) $\sqrt{x} - 4\tan^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$

(e) $4\sqrt{x} + 2\tan^{-1}(\sqrt{x}) + C$

23. The series $\sum_{n=1}^{+\infty} \frac{2^n + (-4)^n}{8^n}$

(a) converges and its sum is 0

(b) converges and its sum is $\frac{2}{3}$

(c) converges and its sum is $\frac{3}{8}$

(d) converges and its sum is $\frac{3}{4}$

(e) diverges

24. The area of the region bounded by the curves

$$y = 5x - x^2 \text{ and } y = x$$

is equal to

(a) $\frac{32}{3}$

(b) $\frac{64}{5}$

(c) $\frac{32}{7}$

(d) $\frac{18}{5}$

(e) $\frac{35}{6}$

25. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx =$

(a) $x - \sqrt{1-x^2}\cdot\sin^{-1}x + C$

(b) $\ln|1-x| - \sqrt{1-x^2}\cdot\sin^{-1}x + C$

(c) $\sqrt{1-x^2} - \sin^{-1}x + C$

(d) $\sqrt{1-x^2}(1-\sin^{-1}x) + C$

(e) $\frac{1}{2}(\sin^{-1}x)^2 + C$

26. The length of the curve

$$y = 10 + 2x^{3/2}, \quad 0 \le x \le 1$$

is equal to

(a) $\frac{2}{27}(10\sqrt{10} - 1)$

(b) $\frac{1}{27}(\sqrt{10} - 1)$

(c) $\frac{2}{9}$

(d) $\frac{2}{9}(10\sqrt{10} - 3)$

(e) $\frac{5}{27}(\sqrt{10} - 10)$

27. A power series representation for $f(x) = \dfrac{2}{(1-2x)^2}$ is given by

(a) $\sum\limits_{n=1}^{+\infty} n \cdot 2^n x^{n-1}$

(b) $\sum\limits_{n=0}^{+\infty} 2^n \dfrac{x^{n+1}}{n+1}$

(c) $\sum\limits_{n=0}^{+\infty} 2^n x^n$

(d) $\sum\limits_{n=0}^{+\infty} n \cdot 2^n x^{n+1}$

(e) $\sum\limits_{n=1}^{+\infty} \dfrac{2^n}{n} x^n$

28. The area of the surface obtained by rotating the curve

$$y = x^5, \quad 1 \le x \le 32$$

about the x-axis is given by

(a) $\int_1^{32} 2\pi x^5 \sqrt{1+25x^8}\, dx$

(b) $\int_1^{32} 2\pi x^5 \sqrt{1+5x^4}\, dx$

(c) $\int_1^{2} 2\pi y \sqrt{1+25x^8}\, dy$

(d) $\int_1^{2} 2\pi \sqrt[5]{y} \cdot \sqrt{1+\dfrac{1}{25} y^{-8/5}}\, dy$

(e) $\int_1^{32} 2\pi x \sqrt{1+25x^8}\, dx$