

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

1.) (7pts) Graph the curve and eliminate the parameter to find a Cartesian equation of the curve

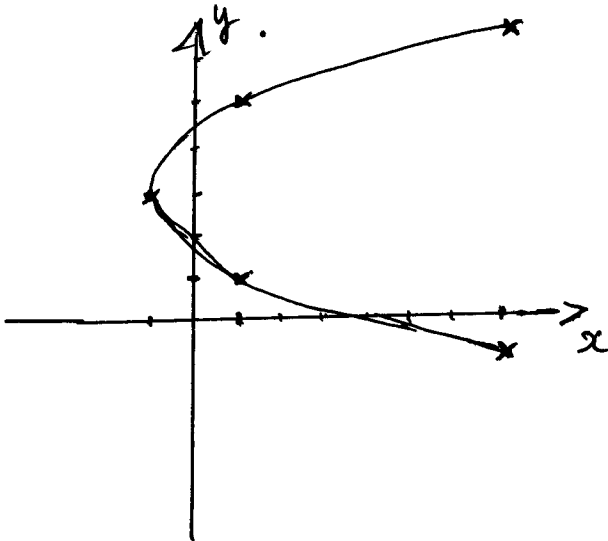
$$\begin{cases} x = 2t^2 - 1 \\ y = 3 - 2t, \quad t \in [-2, 2] \end{cases}$$

2.) (3pts) Find the length of the curve

$$\begin{cases} x = e^{2t} + e^{-2t} \\ y = 1 + 4t, \quad t \in [0, 1] \end{cases}$$

1.)

t	-2	-1	0	1	2
x	7	1	-1	1	7
y	7	5	3	1	-1



From  $y$ , we get  $t = \frac{3-y}{2}$ .  
 We substitute this value in  $x$ , and we find  

$$\left(\frac{3-y}{2}\right)^2 = \frac{x+1}{2}$$

Let now  $x \geq -1$ .

- If  $y \leq 3$ , then  $\frac{3-y}{2} = \sqrt{\frac{x+1}{2}}$ , and  

$$y = 3 - 2\sqrt{\frac{x+1}{2}}$$
- If  $y \geq 3$ , then  $\frac{3-y}{2} = -\sqrt{\frac{x+1}{2}}$ , and  

$$y = 3 + 2\sqrt{\frac{x+1}{2}}$$

2.)

$$\begin{aligned} L &= \int_0^1 \sqrt{(2e^{2t} - 2e^{-2t})^2 + (4)^2} dt \\ &= \int_0^1 \sqrt{4e^{4t} + 4e^{-4t} + 8} dt \\ &= \int_0^1 \sqrt{(2e^{2t} + 2e^{-2t})^2} dt \\ &= \int_0^1 (2e^{2t} + 2e^{-2t}) dt \\ &= [e^{2t} - e^{-2t}]_0^1 \\ &= \frac{e^4 - 1}{e^2} \end{aligned}$$