MATH 201.1 (091)

Quiz 2 (Chap. 10.3-4) Duration: 15mn

Name: ID number:

1.)(5pts) Find all the points of intersection of the polar curves $r = \cos 2\theta + \frac{3}{4}$ and $r = \cos^2\theta$, $0 \le \theta \le \pi$.

2.)(5pts) Find the equation of the tangent line to the curve $r = \sin\theta$ at $\theta = \frac{\pi}{4}$.

1.) $\cos 2\theta + \frac{3}{4} = \cos^2\theta$, $0 \le \theta \le \pi$

$2\cos^2\theta - 1 + \frac{3}{4} = \cos^2\theta$

$\cos^2\theta = 1/4$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1/2$

- $\cos\theta = -\frac{1}{2} = \cos\left(\frac{2}{3}\pi\right)$

$\Rightarrow \begin{cases} \theta = \frac{2}{3}\pi + 2k\pi \\ \text{or} \\ \theta = -\frac{2\pi}{3} + 2k\pi \end{cases}$

$\Rightarrow \boxed{\theta = \frac{2}{3}\pi}$

- $\cos\theta = 1/2 = \cos(\pi/3)$

$\Rightarrow \begin{cases} \theta = \frac{\pi}{3} + 2k\pi \\ \text{or} \\ \theta = -\frac{\pi}{3} + 2k\pi \end{cases}$

$\Rightarrow \boxed{\theta = \frac{\pi}{3}}$

We find two solutions $\theta = \frac{\pi}{3}$; $\theta = \frac{2\pi}{3}$.

2.) $\begin{cases} x = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta \\ y = \sin\theta\sin\theta = \sin^2\theta \end{cases}$

$\frac{dy}{dx} = \frac{2\sin\theta\cos\theta}{\cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta}$

At $\theta = \frac{\pi}{4}$, $\sin 2\theta = \sin\frac{\pi}{2} = 1$

$\cos 2\theta = \cos\frac{\pi}{2} = 0$,

So that there is a vertical tangent of equation $x = \frac{1}{2}$.