MATH 201.1 (091)

Quiz 4 (Chap. 14.7-8) Duration: 15mn

Name: ID number:

1.)(10pts)Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = x^4 + y^4 - 4xy + \sqrt{5}$.

First, we need to find the critical points.

$f_x(x,y) = 4x^3 - 4y$

$f_y(x,y) = 4y^3 - 4x$.

We consider the system

$$\begin{cases} 4x^3 - 4y = 0 \\ 4y^3 - 4x = 0 \end{cases}.$$

$y = x^3$ and $x = y^3$ imply

$x^9 - x = 0$

$\Leftrightarrow x(x^8 - 1) = 0$

$\Leftrightarrow x(x-1)(x+1)(x^2+1)(x^4+1) = 0$

$\Rightarrow x = 0,\ x = -1,\ x = 1$

- if $x = 0$, then $y = 0$
- if $x = -1$, then $y = -1$
- if $x = 1$, then $y = 1$

So, we find the three points $(0,0)$, $(-1,-1)$ and $(1,1)$.

Now, let us do the second derivative test at each point.

$f_{xx}(x,y) = 12x^2$, $f_{yy}(x,y) = 12y^2$

$f_{xy}(x,y) = -4$.

$D = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

- At $(0,0)$, $D = -16$
  $f(0,0) = \sqrt{5}$ is a saddle point
- At $(-1,-1)$, $D = 144 - 16 = 128 > 0$
  $f_{xx}(-1,-1) = 12 > 0$
  $f(-1,-1) = -2 + \sqrt{5} > 0$ is a local minimum
- At $(1,1)$, $D = 128 > 0$
  $f_{xx}(1,1) = -2 + \sqrt{5} > 0$ is a local minimum.