MATH 201.2 (091)

Quiz 2 (Chap. 10.3-4) Duration: 15mn

Name: ID number:

1.)(5pts) Find all the points of intersection of the polar curves $r = \frac{1}{2}\sin 2\theta$ and $r = \cos^2\theta$, $0 \le \theta \le \pi$.

2.)(5pts) Find the equation of the tangent line to the curve $r = \cos\theta$ at $\theta = \frac{\pi}{4}$.

1-) $\frac{1}{2}\sin 2\theta = \cos^2\theta$, $0 \le \theta \le \pi$

$\sin\theta\cos\theta = \cos^2\theta$

$\sin\theta\cos\theta - \cos^2\theta = 0$

$\cos\theta(\sin\theta - \cos\theta) = 0$

$\Rightarrow \cos\theta = 0$ or $\sin\theta = \cos\theta$

- $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + k\pi$

$\boxed{\theta = \pi/2}$

- $\sin\theta = \cos\theta$

$\Leftrightarrow \tan\theta = 1 = \tan\pi/4$

$\Rightarrow \theta = \frac{\pi}{4} + k\pi$

$\boxed{\theta = \pi/4}$

We find two solutions $\theta = \pi/4$ and $\theta = \pi/2$.

2-) $\begin{cases} x = \cos\theta\cos\theta = \cos^2\theta \\ y = \cos\theta\sin\theta = \frac{1}{2}\sin 2\theta \end{cases}$

$\frac{dy}{dx} = -\frac{\cos 2\theta}{2\cos\theta\sin\theta} = -\frac{\cos 2\theta}{\sin 2\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = 0$

At $\theta = \pi/4$, there is an horizontal tangent of equation $y = \frac{1}{2}$.