MATH 201.2 (091)

Quiz 4 (Chap. 14.7-8) Duration: 10mn

Name: ID number:

1.)(10pts)Find the maximum and minimum values of the function $f(x, y, z) = x + 2y - 3z$ subject to the constraint $z = 4x^2 + y^2$.

We use Lagrange multipliers.

We set $g(x,y,z) = z - 4x^2 - y^2$.

We have

$f_x(x,y,z) = 1$, $f_y(x,y,z) = 2$

$f_z(x,y,z) = -3$, so that

$\nabla f = \langle 1, 2, -3 \rangle$.

We have

$g_x(x,y,z) = -8x$, $g_y(x,y,z) = -2y$

and $g_z(x,y,z) = 1$, so that

$\nabla g = \langle -8x, -2y, 1 \rangle$.

Now, we solve the system

$$\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ g(x,y,z) = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 1 = -\lambda 8x & (1) \\ 2 = -2\lambda y & (2) \\ -3 = \lambda \implies \underline{\lambda = -3} \\ z - 4x^2 - y^2 = 0 & (4) \end{cases}$$

(1) $\Rightarrow$ $x = 1/24$

(2) $\Rightarrow$ $y = 1/3$

(4) $\Rightarrow$ $z = \frac{17}{144}$.

$$f\left(\frac{1}{24}, \frac{1}{3}, \frac{17}{144}\right) = \frac{1}{24} + \frac{2}{3} - 3\left(\frac{17}{144}\right)$$

$$= \frac{1}{24} + \frac{2}{3} - \frac{17}{48}$$

$$= 17/48.$$

We find one extreme value.