King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 201
Major Exam I, Semester I, 2009-2010
Duration: 120 minutes

Name: _____

ID: _____

Section: _____

Answer the questions in the space provided. You must show your work or explain your solution otherwise points may be deducted. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation.

1. Write clearly.
2. Show all your steps.
3. No credits will be given to wrong steps.
4. Calculators and mobile phones are NOT allowed in this exam.

Q#	Marks	Maximum Marks
1		16
2		17
3		16
4		17
5		17
6		17
Total		100

Problem1 (16 pts) .

a) Find the rectangular (Cartesian) equation for the polar curve given by

$$r = \sin^2\frac{\theta}{2} + \tan\theta$$

b) A parametric curve is given by the equations

$$x = 2\cos t - 1 \qquad \text{and} \qquad y = 1 + \cos t.$$

Sketch the curve and indicate with an arrow the direction in which it is traced as the parameter increases from 0 to π.

Problem2 (17 pts).

a) A curve C is defined by the parametric equations

$$x = \theta^2 \qquad \text{and} \qquad y = 2(1 - \cos\theta), \qquad 0 \le \theta \le 2\pi.$$

Find (if exist) the points on C where the tangent is horizontal or vertical.

b) For the curve given by

$$x = \sqrt{t} \qquad \text{and} \qquad y = \frac{1}{2}(t^2 - 2), \qquad t \ge 0.$$

Find the slope and concavity at the point $(\sqrt{2}, 1)$.

Problem 3 (16 pts).

a) Sketch the curve with polar equation

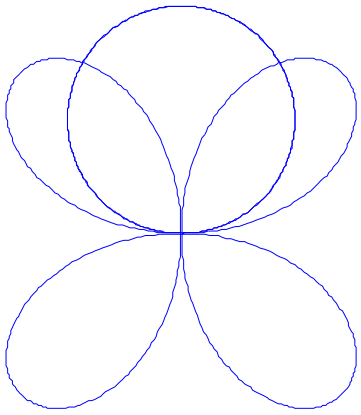
$$r = 2\cos\left(\frac{\theta}{2}\right), \qquad 0 \le \theta \le 2\pi \ .$$

b) Find the equation of the tangent line to the polar curve in part a) at $\theta = \pi/2$

Problem 4 (17 pts).

a) The graphs of the four-leaf rose curve $r = \sin 2\theta$ and the circle $r = \sin\theta$ are drawn below for you.

Find the area of the shaded region inside both curves as a sum of two definite integrals. (**DO NOT CALCULATE** the integrals)



b) Find the length of the parametric curve

$$x = \sin t - t\cos t \qquad \text{and} \qquad y = \cos t + t\sin t, \qquad 0 \le t \le \pi.$$

Problem 5 (17 pts)

a) Let

$$\vec{a} = \langle -2, 2, 4 \rangle, \quad \text{and} \quad \vec{b} = \langle 3, 3, -1 \rangle.$$

Find a vector $\vec{v}$ with length 4 that has the same direction as

$$\vec{b} - \frac{1}{2}\vec{a}.$$

b) Consider the vectors

$$\vec{a} = \langle -3, 4, 12 \rangle, \quad \text{and} \quad \vec{b} = \langle 24, 8, 6 \rangle.$$

Calculate the cosine of the angle between $\vec{a}$ and $\vec{b}$, and find the vector projection of $\vec{a}$ on $\vec{b}$.

Problem 6 (17 pts).

a) Find a unit vector that is orthogonal to both

$$\vec{u} = \vec{i} - 4\vec{j} + \vec{k} \qquad \text{and} \qquad \vec{v} = 2\vec{i} + 3\vec{j}.$$

b) Consider the vectors

$$\vec{u} = \langle 3, 2, x \rangle, \quad \text{and} \quad \vec{v} = \langle 2x, 4, x \rangle.$$

Find (if exist) the values of x such that $\vec{u}$ and $\vec{v}$ are orthogonal and the values of x such that $\vec{u}$ and $\vec{v}$ are parallel.