

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201

Major Exam II, Semester I, 2009-2010

Duration: 120 minutes

Name: _____

ID: _____

Section: _____

Answer the questions in the space provided. You must show your work or explain your solution otherwise points may be deducted. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation.

1. Write clearly.
2. Show all your steps.
3. No credits will be given to wrong steps.
4. Calculators and mobile phones are NOT allowed in this exam.

Q#	Marks	Maximum Marks
1		16
2		17
3		16
4		17
5		17
6		17
Total		100

Problem1 (16 pts).

Consider the surface given by the equation

$$y^2 = 4x^2 + 9z^2$$

a) Identify the surface and find the traces in the planes $x = k$, $y = k$, and $z = k$ to sketch the graph.

b) Write the equation

$$4z^2 = x^2 + y^2$$

in spherical coordinates

Problem2 (17 pts).

a) The directional derivative of $f(x,y)$ at $(1, 1)$ is $\sqrt{2}$ in the direction of $\vec{u}_1 = \vec{i}$, and it is -3 in the direction of $\vec{u}_2 = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$.

Find the directional derivative of f at $(1, 1)$ in the direction of

$$\vec{u}_3 = \frac{2}{\sqrt{13}}\vec{i} + \frac{3}{\sqrt{13}}\vec{j}.$$

b) Determine an equation of the tangent plane to

$$f(x,y) = 3e^{(x-y)} \ln x$$

at the point $(1, 1)$.

Problem 3 (16 pts).

a) Find the equation of the tangent plane and a set of parametric equations of the normal line to the surface

$$xz - yz^3 + yz^2 = 2$$

at the point $(2, -1, 1)$.

b) The surface of a mountain is modeled by the equation

$$h(x, y) = 4000 - 0.001x^2 - 0.004y^2.$$

Suppose that a mountain climber is at the point $(500, 300, 3390)$ on the mountain. In what direction should the climber move in order to climb at the greatest rate?

Problem 4 (17 pts).

a) Find the linear approximation of

$$f(x,y) = \frac{y}{x}$$

at $(3, 6)$ and use it to approximate $f(3.03, 5.97)$.

b) Find and sketch the domain of the function

$$f(x,y) = \sqrt{1 + x - y^2}$$

Problem 5 (17 pts)

a) Consider the function

$$f(x,y) = \frac{x^2y^2}{x^2 + 2y^2}$$

Show that if $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ exists, it must be 0. Prove that the limit is in fact equal to 0.

b) Show that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2}$$

does not exist.

Problem 6 (17 pts).

a) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ when $s = 3$ and $t = \frac{\pi}{4}$ for the function given by

$$w = x^2 - y^2$$

where

$$x = s \cos t, \quad y = s \sin t$$

b) Find the equation of the plane \mathfrak{R}_1 passing through the points $(0, 0, 0)$, $(1, 2, 3)$, $(-2, 3, 3)$, and the distance between the plane \mathfrak{R}_1 and the plane \mathfrak{R}_2 whose equation is given by

$$-3x - 9y + 7z = 4$$