MATH 201.5 (091)

Quiz 2 (Chap. 10.3-4) Duration: 15mn

Name: ID number:

1.)(5pts) Find all the points of intersection of the polar curves $r = \tan\theta$ and $r = \cot\theta$, $0 < \theta < \frac{\pi}{2}$.

2.)(5pts) Find the equation of the tangent line to the curve $r = \tan\theta$ at $\theta = 0$.

1.) $\tan\theta = \cot\theta$, $0 < \theta < \frac{\pi}{2}$

$\Leftrightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\sin\theta}$

$\Leftrightarrow \sin^2\theta = \cos^2\theta$ (1)

First method to solve (1).

$\sin\theta = \cos\theta$ or $\sin\theta = -\cos\theta$

- $\sin\theta = \cos\theta = \sin(\theta + \frac{\pi}{2})$

$\begin{cases} \theta = \theta + \frac{\pi}{2} + 2k\pi \\ \text{or } \theta = \pi - (\theta + \frac{\pi}{2}) + 2k\pi \end{cases}$, $k \in \mathbb{Z}$

This is $2\theta = \frac{\pi}{2} + 2k\pi$

$\theta = \frac{\pi}{4} + k\pi$

$\boxed{\theta = \pi/4}$

- $\sin\theta = -\cos\theta = \sin(\theta - \frac{\pi}{2})$

$\begin{cases} \theta = \theta - \frac{\pi}{2} + 2k\pi \\ \theta = \pi - (\theta - \frac{\pi}{2}) + 2k\pi \end{cases}$

This is $2\theta = \frac{3\pi}{2} + 2k\pi$

$\theta = \frac{3\pi}{4} + k\pi$

There is no solution here, because $0 < \theta < \pi/2$.

Second method to solve (1)

$\cos^2\theta = 1/2$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

Since $0 < \theta < \frac{\pi}{2}$, we have to solve only $\cos\theta = \sqrt{2}/2 \Rightarrow \boxed{\theta = \pi/4}$

2.) $\begin{cases} x = \tan\theta\cos\theta = \sin\theta \\ y = \tan\theta\sin\theta = \frac{\sin^2\theta}{\cos\theta} \end{cases}$

$\frac{dy}{dx} = \frac{2\sin\theta\cos^2\theta + \sin^3\theta}{\cos^3\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=0} = 0$

At $\theta = 0$, there is an horizontal tangente of equation

$\boxed{y = 0}$.

1.) Again:

Another way to solve (1)

$\cos^2\theta = \sin^2\theta \Rightarrow \cos^2\theta - \sin^2\theta = 0$

$\cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} + k\pi$

$\theta = \pi/4 + \frac{k\pi}{2}$

$\boxed{\theta = \pi/4}$, since $0 < \theta < \pi/2$