

Name: _____

ID number: _____

- 1.) (5pts) Find the point of intersection of the plane $2x + y - z = 0$ and the line through $P(3, 1, 0)$ that is perpendicular to the plane.
 2.) (5pts) Let $f(x, y, z) = x^2 + y^2 - z^2$. Find and sketch the level surface of f that passes through the point $Q(1, -1, 1)$.

1.) let $I(x_0, y_0, z_0)$ this point.

let \vec{n} a normal vector to the plane $2x + y - z = 0$.

We have

$$\begin{cases} 2x_0 + y_0 - z_0 = 0 \\ \vec{PI} \times \vec{n} = \vec{0} \end{cases}$$

$$\vec{PI} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_0 - 3 & y_0 - 1 & z_0 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \langle -y_0 - z_0 + 1, x_0 + 2z_0 - 3, x_0 - 2y_0 - 1 \rangle$$

We solve

$$\begin{cases} 2x_0 + y_0 - z_0 = 0 & (1) \\ -y_0 + z_0 + 1 = 0 & (2) \\ x_0 + 2z_0 - 3 = 0 & (3) \\ x_0 - 2y_0 - 1 = 0 & (4) \end{cases}$$

$$I\left(\frac{2}{3}, -\frac{1}{6}, \frac{7}{6}\right)$$

2.) level surfaces are $x^2 + y^2 - z^2 = k, k \in \mathbb{R}$.

The one that contains Q satisfies $1^2 + (-1)^2 - 1^2 = k$, that is $k = 1$,

so we have $x^2 + y^2 - z^2 = 1$.

This is an hyperboloid of one sheet.

