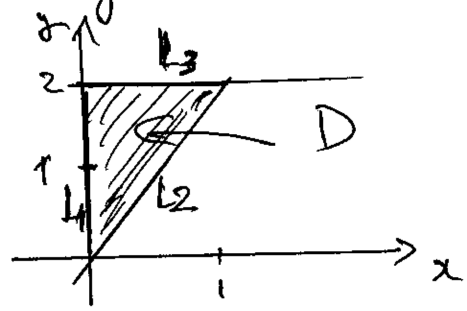
MATH 201.5 (091)

Quiz 4 (Chap. 14.7-8) Duration: 20mn

Name: ID number:

1.)(10pts)Find the absolute maximum and minimum values of $f(x,y) = 2x^2 - 4x + y^2 - 2y + 1$ in the closed triangular region D bounded by $x = 0$, $y = 2$ and $y = 2x$ in the first quadrant.

First, we sketch the region D.



We look for maximum and minimum values of f on the boundary of D.

(L_1): $\begin{cases} x=0, \\ 0 \le y \le 2, \end{cases}$ $f(x,y) = y^2 - 2y + 1$.

y	0	1	2
f(x,y)	1 ↘	0	↗ 1

$f(0,1) = 0$ a minimum value

$f(0,0) = f(0,2) = 1$ max. values.

(L_2): $\begin{cases} 0 \le x \le 1, \; 0 \le y \le 2 \\ y = 2x, \end{cases}$ $f(x,y) = 6x^2 - 8x + 1$.

x	0	2/3	1
f(x,y)	1 ↘	$-5/3$	↗ -1

$f\left(\frac{2}{3}, \frac{4}{3}\right) = -5/3$ is a minimum value

$f(0,0) = 1$ is a maximum value

(L_3): $0 \le x \le 1$, $y = 2$

$f(x,y) = 2x^2 - 4x + 1$

x	0	1
f(x,y)	1 →	-1

$f(1,2) = -1$ a minimum value

$f(0,2) = 1$ a maximum value

Now, the critical points of f inside D.

$\begin{cases} f_x(x,y) = 0 \\ f_y(x,y) = 0 \end{cases} \iff \begin{cases} 4x - 4 = 0 \\ 2y - 2 = 0 \end{cases}$

$\Rightarrow x = 1, \; y = 1$

$(1,1)$ is not inside D, so it is rejected.

Thus, the absolute maximum of f is $f(0,0) = f(0,2)$.

The absolute minimum is $f\left(\frac{2}{3}, \frac{4}{3}\right) = -5/3$.