KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Major Exam 1

Math 301
(Methods of Applied Mathematics)

Time Allowed: 1 ¼ Hours

Student Name:_______________       Id. No._____________

Section:_______________

Note

No programmable calculators and mobile phones allowed in the examination hall. For all questions show calculations in support of your answers.

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Instructor Name
Ashfaque H. Bokhari
Q 1. Evaluate \( \int_C x \, y \, ds \), where \( C \) consists of the straight line segment \( C_1 \) from \((-3, 3)\) to \((0, 0)\) followed by a curve defined by \( y = \frac{x^4}{16} \):
Q2. Use independence of path in integrals to evaluate \[
\int_{(1,1)}^{(2,2)} (e^x \sin y - y + 1) dx + (e^x \cos y - x + 2) dy
\]
Q3. Use Green’s theorem to evaluate
\[ \oint_C \left( x \sin x + y \right) \, dx - \left( e^{y^2} + x \right) \, dy \]
over the closed curve drawn below:

\[ \begin{array}{c}
(1, 5) \\
(1, -1) \\
(-1, -1)
\end{array} \]
Q4.

(a): Express the vector \( \vec{r}(t) = (2 - 3t, 1 + t, -4t) \) in terms of arc length parameter ‘s’ measured from the point where \( t=0 \) in the direction of increasing \( t \).

(b): Find magnitude of the tangent vector to any point on the resultant vector \( \vec{r}(s) \).
Q5. Evaluate the surface integral \( \iint_S G(x, y, z) \, ds \), where \( G = x^2 + y^2 + z^2 \) and \( S \) is portion of the plane \( z = x + 1 \) that lies inside the cylinder \( x^2 + y^2 = 1 \).
Q6. Use Stokes theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (2z + x) \hat{i} + (y - z) \hat{j} + (x + y) \hat{k}$ where $C$ is a counter clock-wise oriented triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. 