KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Final Exam
Math 301
Methods of Applied Mathematics

Time Allowed: 3 Hours

Student Name:_______________  ID. No._______________
Section No:________________

Note
No programmable calculators and mobile phones allowed in the examination hall.
For all questions show calculations in support of your answers.

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Q1. Use Laplace Transform to solve the following Boundary value problem: (7 points)

\[ a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}, \quad 0 < x < 2, \quad t > 0 \]

\[ u(0,t) = 0, \quad u(2,t) = 0 \quad t > 0 \]

\[ u(x,0) = 0, \quad \left. \frac{\partial u(x,t)}{\partial t} \right|_{t=0} = \sin 2\pi x, \quad 0 < x < 2 \]
Q2. Use an appropriate Fourier transform to solve the boundary value problem:

\[ k \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}, \]

\[ u(0,t) = 1, \quad t > 0 \quad \text{(6 points)} \]

\[ u(x,0) = 0, \quad 0 < x < \infty \]
Q3. Solve the following boundary value problem: (7 points)

\[ \frac{\partial^2 u(r, z)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, z)}{\partial r} + \frac{\partial^2 u(r, z)}{\partial z^2} = 0, \quad 0 < z < 1, \quad 0 < r < 4 \]

\[ u(4, z) = 0, \quad 0 < z < 1 \]

\[ u(r, 0) = 0, \quad u(r, 1) = 1, \quad 0 < r < 4 \]
Q4. Solve the Ditchlet boundary valued problem given by, (7 points)

\[ \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0, \quad 0 < x < 1, \quad 0 < y < 2 \]

\[ u(0, y) = 0, \quad u(1, y) = 0, \]

\[ u(x, 0) = 0, \quad u(x, 2) = x. \]
Q5. Expand \( f(x) = x^2 \), \( 0 < x < 2 \) in the Fourier Bessel series using Bessel functions of order 2 that satisfy the boundary condition \( J_2(2\alpha) = 0 \). (6 points)
Q6. Use the divergence theorem to evaluate \( \iint_S \vec{F} \cdot \vec{n} \, ds \) when \( \vec{F} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{x^2 + y^2 + z^2} \) with \( D \) a region bounded by two concentric spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 16 \) forming surface \( S \).

(7 points)