- KFUPM SEM II (Term 092) Name: KEY Serial #: ______

MATH 101-1-4-10 Quiz # 1 ID: #: ______ Sec. #: ______

1. (3-points) Let $f(x) = \begin{cases} 3a + x & \text{if } x < -1 \\ 4x^2 + a & \text{if } x > -1 \end{cases}$. Find the value of a for which $\lim_{x \to -1} f(x)$ exists.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (3a + x) = 3a - 1 \quad (i)$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (4x^2 + a) = 4 + a \quad (ii)$$

$$(i) = (ii) \Rightarrow 3a - 1 = 4 + a \Rightarrow 2a = 5 \Rightarrow$$

$$\boxed{a = \frac{5}{2}}$$

2. (3-points) If $f(x) = [2x - 1] + [2x + 1]$, find $\lim_{x \to 1^-} f(x)$, where $[y]$ is the greatest integer less than or equal to y.

$$\lim_{x \to 1^-} [\![2x-1]\!] + \lim_{x \to 1^+} [\![2x+1]\!]$$

$$= 0 + 2 = 2$$

Trial values

$x \to 1^-$, say, $x = 0.9$

$[\![2x-1]\!] = [\![0.8]\!] = 0$

$[\![2x+1]\!] = [\![2.8]\!] = 2$

P.T.O

3. (4-points) Evaluate $\lim_{h \to 0} \frac{(5+h)^{-1} - 5^{-1}}{3h}$. (% type)

$$= \lim_{h \to 0} \frac{1}{3h}\left[\frac{1}{5+h} - \frac{1}{5}\right]$$

$$= \lim_{h \to 0} \left[\frac{1}{3\cancel{h}} \frac{\overset{-1}{\cancel{(5-5-h)}}}{5(5+h)}\right]$$

$$= \lim_{h \to 0} \frac{-1}{15(5+h)} = -\frac{1}{75}$$

4. (5-points) If $\frac{x^2-3x+2}{x^2-5x+6} \le f(x) \le \frac{x^2-4}{x^2-8x+12}$, find $\lim_{x \to 2} f(x)$.

$$\lim_{x \to 2} \frac{x^2-3x+2}{x^2-5x+6} = \lim_{x \to 2} \frac{(x-1)(x-2)}{(x-2)(x-3)}$$

$$= \lim_{x \to 2} \frac{x-1}{x-3} = -1$$

and $$\lim_{x \to 2} \frac{x^2-4}{x^2-8x+12} = \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x-6)}$$

$$= \lim_{x \to 2} \frac{x+2}{x-6} = \frac{4}{-4} = -1$$

$$\Rightarrow \lim_{x \to 2} f(x) = -1$$

P.T.O $\longrightarrow$

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MATH 101-1-4-10 Quiz # 1 ID: #: ____________ Sec. #: ________

1. (3-points) Let $f(x) = \begin{cases} x - 3a & \text{if } x < -1 \\ 3x^2 + a & \text{if } x > -1 \end{cases}$. Find the value of a for which $\lim_{x \to -1} f(x)$ exists.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x - 3a) = -1 - 3a \quad (i)$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (3x^2 + a) = 3 + a \quad (ii)$$

$$(i) = (ii) \Rightarrow -1 - 3a = 3 + a \Rightarrow 4a = -4 \Rightarrow$$

$$\boxed{a = -1}$$

2. (3-points) If $f(x) = [3x - 1] + [2x + 1]$, find $\lim_{x \to 1^-} f(x)$, where $[y]$ is the greatest integer less than or equal to y.

$$\lim_{x \to 1^-} [\![3x-1]\!] + \lim_{x \to 1^-} [\![2x+1]\!]$$

$$= 1 + 2 = 3.$$

Trial values

$x \to 1^-$, say, $x = 0.9$

$[\![3x-1]\!] = [\![1.7]\!] = 1$

$[\![2x+1]\!] = [\![2.8]\!] = 2$

P.T.O $\longrightarrow$

3. (4-points) Evaluate $\lim_{h \to 5} \frac{(10-h)^{-1} - 5^{-1}}{4(h-5)}$. (0/0 type)

$$= \lim_{h \to 5} \frac{1}{4(h-5)} \left[\frac{1}{10-h} - \frac{1}{5} \right]$$

$$= \lim_{h \to 5} \left[\frac{1}{\cancel{4(h-5)}} \frac{\cancel{(5-10+h)}}{5(10-h)} \right]$$

$$= \lim_{h \to 5} \left[\frac{1}{20(10-h)} \right]$$

$$= \frac{1}{100}$$

4. (5-points) If $\frac{x^2+2x-3}{x^2-1} \le f(x) \le \frac{x^2-1}{x^2-x}$, find $\lim_{x \to 1} f(x)$.

$$\lim_{x \to 1} \frac{x^2+2x-3}{x^2-1} = \lim_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{x+3}{x+1} = \frac{4}{2} = 2$$

$$\lim_{x \to 1} \frac{x^2-1}{x^2-x} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x(x-1)}$$

$$= \lim_{x \to 1} \frac{x+1}{x} = 2.$$

P.T.O

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MATH 101-1-4-10 Quiz # 1 ID: #: ______ Sec. #: ______

1. (3-points) Let $f(x) = \begin{cases} \frac{a}{3} + \sin x & \text{if } x < \frac{\pi}{4} \\ -a + 2\cos 2x & \text{if } x > \frac{\pi}{4} \end{cases}$. Find the value of a for which $\lim_{x\to\pi/6} f(x)$ exists.

$$\lim_{x\to(\frac{\pi}{4})^-} f(x) = \lim_{x\to(\frac{\pi}{4})^-}\left(\frac{a}{3} + \sin x\right) = \frac{a}{3} + \frac{\sqrt{2}}{2} \quad (i)$$

$$\lim_{x\to(\frac{\pi}{4})^+} f(x) = \lim_{x\to(\frac{\pi}{4})^+}(-a + 2\cos 2x) = -a \quad (ii)$$

$$(i) = (ii) \Rightarrow \frac{a}{3} + \frac{\sqrt{2}}{2} = -a \Rightarrow \frac{4}{3}a = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \boxed{a = \frac{3\sqrt{2}}{8}}$$

2. (3-points) If $f(x) = [1-2x] + [1-3x]$, find $\lim_{x\to1^-} f(x)$, where $[y]$ is the greatest integer less than or equal to y.

$$\lim_{x\to1^-}\left([\![1-2x]\!] + [\![1-3x]\!]\right)$$

$$= \lim_{x\to1^-}[\![1-2x]\!] + \lim_{x\to1^-}[\![1-3x]\!]$$

$$= -1 - 2 = -3$$

trial values

$x \to 1^-$, say, $x = 0.9$

$\Rightarrow [\![1-2x]\!] = [\![-0.8]\!] = -1$

$[\![1-3x]\!] = [\![-1.7]\!] = -2$

P.T.O $\rightarrow$

3. (4-points) Evaluate $\lim_{h\to 1} \frac{(4+h)^{-1}-5^{-1}}{3(h-1)}$. ($\frac{0}{0}$ type)

$$= \lim_{h\to 1} \frac{1}{3(h-1)}\left[\frac{1}{4+h}-\frac{1}{5}\right]$$

$$= \lim_{h\to 1}\left[\frac{1}{3(h-1)}\ \frac{5-(4+h)}{5(4+h)}\right]$$

$$= \lim_{h\to 1}\left[\frac{1}{3(h-1)}\ \frac{1-h}{5(4+h)}\right]$$

$$= \lim_{h\to 1}\left[\frac{-1}{15(4+h)}\right] = -\frac{1}{75}$$

4. (5-points) If $\frac{x^2+2x-3}{x^2-9} \le f(x) \le \frac{x^2+4x+3}{x^2+3x}$, find $\lim_{x\to -3} f(x)$.

$$\lim_{x\to -3}\left(\frac{x^2+2x-3}{x^2-9}\right) = \lim_{x\to -3} \frac{(x+3)(x-1)}{(x+3)(x-3)}$$

$$= \lim_{x\to -3} \frac{x-1}{x-3} = \frac{-4}{-6} = \frac{2}{3}$$

and $$\lim_{x\to -3} \frac{x^2+4x+3}{x^2+3x} = \lim_{x\to -3} \frac{(x+1)(x+3)}{x(x+3)}$$

$$= \lim_{x\to -3} \frac{x+1}{x} = \frac{-2}{-3} = \frac{2}{3}$$

$\Rightarrow \lim_{x\to -3} f(x) = \frac{2}{3}$ by the Squeeze Theorem

P.T.O $\longrightarrow$

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MATH 101-1-4-10 Quiz # 1 ID: #: ______ Sec. #: ______

1. (3-points) Let $f(x) = \begin{cases} \frac{4}{3}a + 3\tan x & \text{if } x < \frac{\pi}{4} \\ a + \sqrt{2}\sin x & \text{if } x > \frac{\pi}{4} \end{cases}$. Find the value of a for which $\lim_{x\to\pi/4} f(x)$ exists.

$$\lim_{x\to(\frac{\pi}{4})^-} f(x) = \lim_{x\to(\frac{\pi}{4})^-} \left(\tfrac{4}{3}a + 3\tan x\right) = \tfrac{4}{3}a + 3 \quad (i)$$

$$\lim_{x\to(\frac{\pi}{4})^+} f(x) = \lim_{x\to(\frac{\pi}{4})^+} \left(a + \sqrt{2}\sin x\right) = a + 1 \quad (ii)$$

$$(i) = (ii) \Rightarrow \tfrac{4}{3}a + 3 = a + 1 \Rightarrow \tfrac{1}{3}a = -2 \Rightarrow$$

$$\boxed{a = -6}$$

2. (3-points) If $f(x) = [5x-1] + [1-5x]$, find $\lim_{x\to1^-} f(x)$, where $[y]$ is the greatest integer less than or equal to y.

$$\lim_{x\to1^-}\left([\![5x-1]\!] + [\![1-5x]\!]\right)$$

$$= \lim_{x\to1^-}[\![5x-1]\!] + \lim_{x\to1^-}[\![1-5x]\!]$$

$$= 3 - 4 = -1$$

trial values
$x\to1^-$, say, $x = 0.9 \Rightarrow$
$[\![5x-1]\!] = [\![3.5]\!] = 3$
$[\![1-5x]\!] = [\![-3.5]\!] = -4$

P.T.O

= 3. (4-points) Evaluate $\lim_{h\to 2} \frac{(3+h)^{-1} - 5^{-1}}{2(h-2)}$. $\left(\frac{0}{0} \text{ type}\right)$

$$= \lim_{h\to 2} \frac{1}{2(h-2)} \left[\frac{1}{3+h} - \frac{1}{5}\right]$$

$$= \lim_{h\to 2} \left[\frac{1}{2(h-2)} \cdot \frac{5-(3+h)}{5(3+h)}\right]$$

$$= \lim_{h\to 2} \left[\frac{1}{2\cancel{(h-2)}} \cdot \frac{\overset{-1}{\cancel{(2-h)}}}{5(3+h)}\right]$$

$$= \lim_{h\to 2} \frac{-1}{10(3+h)} = \frac{-1}{50}$$

4. (5-points) If $\frac{x^2+x}{x^2-1} \le f(x) \le \frac{x^2+3x+2}{x^2+4x+3}$, find $\lim_{x\to -1} f(x)$.

$$\lim_{x\to -1} \frac{x^2+x}{x^2-1} \left(\frac{0}{0} \text{ type}\right) = \lim_{x\to -1} \frac{x(x+1)}{(x-1)(x+1)}$$

$$= \lim_{x\to -1} \frac{x}{x-1} = \frac{-1}{-2} = \frac{1}{2}.$$

and $$\lim_{x\to -1} \frac{x^2+3x+2}{x^2+4x+3} \left(\frac{0}{0} \text{ type}\right) = \lim_{x\to -1} \frac{(x+1)(x+2)}{(x+1)(x+3)}$$

$$= \lim_{x\to -1} \frac{x+2}{x+3} = \frac{1}{2}$$

$\Rightarrow \lim_{x\to -1} f(x) = 1/2$ by the Squeeze Theorem.

P.T.O $\longrightarrow$