• KFUPM	SEM II (Term 092)	Name:	Serial #:
MATH 101-1-4-10	Quiz # 5	ID: #: KEY	Sec. #:

1. (7-points) Given $f(x) = x(1-3x)^{2/3}$, $0 \le x \le 3$.

(a) Find all critical numbers of f.

2

$$f'(x) = (1-3x)^{2/3} + \frac{2}{3}x(1-3x)^{-1/3}(-3)$$

$$= (1-3x)^{-1/3}\left[1-3x-2x\right] = \frac{1-5x}{(1-3x)^{1/3}}$$

$\Rightarrow f'(\frac{1}{5}) = 0$ and $f'(\frac{1}{3})$ DNE and both

1 $\frac{1}{5}, \frac{1}{3} \in \operatorname{dom} f \Rightarrow$

1 $\frac{1}{5}$ and $\frac{1}{3}$ are the critical points of f on $[0,3]$

(b) Find the absolute maximum and the absolute minimum of f.

$f(0) = 0$

$f(\frac{1}{5}) = \frac{1}{5}(-\frac{2}{5})^{2/3} = \frac{1}{5}\sqrt[3]{\frac{4}{25}}$

$f(\frac{1}{3}) = \frac{1}{3}(0) = 0$) Absolute minimum

$f(3) = 3(-8)^{2/3} = 3(4) = 12$) Absolute maximum

$f(0) = f(\frac{1}{3}) = 0$ is the absolute minimum value of f.

$f(3) = 12$ is the absolute maximum value of f.

• 2. (8-points) Given $f(x) = \dfrac{2x+3}{3x-4}$, $2 \le x \le 3$,

(a) Is the Mean Value Theorem applicable to f? Why? (You must give reasons)

$f(x)$ is discontinuous at $x = 4/3 \notin [2,3]$,

and since f is a rational function $\Rightarrow$

f is continuous on $[2,3]$ and differentiable on $(2,3)$

Also $f(2) = \frac{7}{2} \neq f(3) = \frac{9}{5}$

$\Rightarrow$ The Mean Value Theorem is applicable to f on $[2,3]$.

(b) If your answer is yes to Part (a), then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

From (a) $\Rightarrow$ there exists $c \in (2,3)$ such that

$$f'(c) = \frac{f(3) - f(2)}{3-2} = \frac{9}{5} - \frac{7}{2} = \frac{-17}{10} \quad (i)$$

$$f'(x) = \frac{2(3x-4) - 3(2x+3)}{(3x-4)^2} = \frac{-17}{(3x-4)^2} \quad (ii)$$

$$(i) \& (ii) \Rightarrow \frac{-17}{(3c-4)^2} = -\frac{17}{10} \Rightarrow (3c-4)^2 = 10 \Rightarrow$$

$3c - 4 = \sqrt{10}$ or $3c = -4 - \sqrt{10}$ $\Rightarrow$

$$\underbrace{c = \frac{4+\sqrt{10}}{3} \in (2,3)}_{\text{The required } c} \quad \text{or} \quad \underbrace{c = \frac{-4-\sqrt{10}}{3} \notin (2,3)}_{\text{Rejected.}}$$

KFUPM	SEM II (Term 092)	Name: KEY	Serial #:
MATH 101-1-4-10	Quiz # 5	ID: #:	Sec. #:

1. (7-points) Given $f(x) = x(2x-2)^{2/3}$, $-3 \le x \le 0$.

(a) Find all critical numbers of f.

$$f'(x) = (2x-2)^{2/3} + \frac{2}{3}x(2x-2)^{-1/3}(2)$$

$$= \frac{1}{3}(2x-2)^{-1/3}\left[3(2x-2)+4x\right] = \frac{10x-6}{3(2x-2)^{1/3}}$$

$\Rightarrow f'(\frac{3}{5}) = 0$ and $f'(1)$ DNE

But $\frac{3}{5}, 1 \notin \text{dom } f = [-3,0] \Rightarrow$

f has no critical pts on $(-3,0)$.

(b) Find the absolute maximum and the absolute minimum of f.

$f(-3) = (-3)(-8)^{2/3} = -3(4) = -12$ The absolute minimum

$f(0) = 0$ The absolute maximum.

$\Longrightarrow f(-3) = -12$ is the absolute minimum value of f

and $f(0) = 0$ is the absolute maximum value of f.

2. (8-points) Given $f(x) = \dfrac{3x-4}{2x+3}$, $\quad -1 \le x \le 0$,

(a) Is the Mean Value Theorem applicable to f? Why? (You must give reasons)

f is discontinuous at $x = -3/2 \notin [-1, 0]$,

and since f is a rational function $\Rightarrow$

f is continuous on $[-1,0]$ and differentiable on $(-1,0)$

Also $f(-1) = -7 \neq f(0) = -\frac{4}{3}$

$\Rightarrow$ The Mean Value Theorem is applicable to f on $[-1, 0]$

(b) If your answer is yes to Part (a), then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

From (a) $\Rightarrow$ there exists $c \in (-1,0)$ such that

$$f'(c) = \frac{f(0) - f(-1)}{0-(-1)} = \frac{-\frac{4}{3}+7}{1} = \frac{17}{3} \quad (i)$$

$$f'(x) = \frac{3(2x+3) - 2(3x-4)}{(2x+3)^2} = \frac{17}{(2x+3)^2} \quad (ii)$$

$$(i) \,\&\, (ii) \Rightarrow \frac{17}{(2c+3)^2} = \frac{17}{3} \Rightarrow (2c+3)^2 = 3$$

$2c+3 = \sqrt{3}$ or $2c+3 = -\sqrt{3} \Rightarrow$

$c = \dfrac{\sqrt{3}-3}{2} \in (-1,0)$ $\qquad$ $c = \dfrac{-3-\sqrt{3}}{2} \notin (-1,0)$

The required c $\qquad$ Rejected.