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Math 101-092

Take-home exam 092

Name : = ----- ID# -----

Sec #: -----

Q 1

- a) Using (ε, δ) -definition prove that $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (1 - 5x) = -14$. Find δ when $\varepsilon = 0.5$.
- b) Given that $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{2}{x+1} = 1/2$ and $\varepsilon = 0.25$. Find the maximum value of δ such that $|f(x) - 1/2| < 0.25$ when $|x - 3| < \delta$

Q # 2 Let $f(x) = \frac{x^2 - 5x + 6}{(x+1)(x-2)}$

1. Evaluate $\lim_{x \rightarrow 2} f(x)$ and $\lim_{x \rightarrow -1} f(x)$
2. Find all the vertical asymptotes (VA) of the function
3. Find all HA of the function

Q# 3 Let $f(x) = \frac{|x^2 - 16|}{x - 4}$

Evaluate $f(4^+)$ and $f(4^-)$

Q# 4 Evaluate the following limits

1. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2} - x)$

2. $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + 5}}{2x + 3}$

3. $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 + 5}}{2x + 3}$

Q# 5

Let $s(t) = 6t^2 + 1$ $2 \leq t \leq 2.5$ be the distance function of a moving particle along a straight line.

1. Find the average velocity of the particle in the interval
2. Find the instantaneous velocity of the particle at $t = 2$

Q # 6 Let $f(x) = [\cos x] + [\sin x]$

1. Plot the function
2. Evaluate $f(0), f(0^+), f(0^-)$
3. Evaluate $f(\frac{\pi}{2}), f(\frac{\pi}{2}^+), f(\frac{\pi}{2}^-)$

Q # 7 Let $f(x) = a[x] + b[-x]$ Find the values of a, b such that $f(x)$ is continuous at $x = 4$.

Q # 8 Let $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$

1. Find the equation of the tangent line at $x = 4$
2. Find the equation of the normal line at $x = 4$

Q # 9 Let $C: f(x) = x^3 + 3$ and $L: y = 9x - 5$

1. Find two points on C where the tangent line is parallel to L
2. Find the distance between these points.

Q# 10 Plot a graph of a function in $0 \leq x \leq 5$ that satisfies the following properties

1. $f(0) = 3, f(0^+) = 5, f(0^-) = -\infty$
2. $f(x)$ ($0 < x < 3$) is continuous
3. $f(3^-) = \infty, f(3^+) = -\infty$
4. $f(x)$ ($3 < x \leq 5$) is continuous.