Math 102
Exam I
Fall
Wednesday 31/03/2010
Net Time Allowed: 120 minutes
1. \[ \int \frac{e^{2x}}{1 + e^{4x}} \, dx = \]

(a) \[ \frac{1}{2} \tan^{-1}(e^{2x}) + c \]

(b) \[ \tan^{-1}(e^{2x}) + c \]

(c) \[ \frac{1}{4} \tan^{-1}(e^{2x}) + c \]

(d) \[ \frac{1}{2} \tan^{-1}(e^{4x}) + c \]

(e) \[ \tan^{-1}(e^{4x}) + c \]

2. Using four rectangles and right endpoints, the area under the graph of

\[ f(x) = \sin x \]

from \( x = 0 \) to \( x = \pi \) is approximately equal to

(a) \[ \frac{\pi(1 + \sqrt{2})}{4} \]

(b) \[ \frac{\sqrt{2}(1 + \pi)}{4} \]

(c) \[ \frac{\pi}{4} \]

(d) \[ \frac{\pi(1 - \sqrt{2})}{2} \]

(e) \[ \pi(1 + \sqrt{2}) \]
3. \[ \int_0^{\frac{1}{2}} \left( \frac{6}{\sqrt{1-t^2}} + \frac{12t-2}{3\sqrt{t}} \right) dt = \]

(a) \( \pi \)

(b) \( \pi + \sqrt{2} \)

(c) \( \pi + 2\sqrt{2} \)

(d) \( \pi + 3\sqrt{2} \)

(e) \( \pi + 4\sqrt{2} \)

4. \[ \int \frac{(x-2)^3}{x^2} dx = \]

(a) \[ \frac{x^2}{2} - 6x + 12 \ln |x| + \frac{8}{x} + c \]

(b) \[ \frac{x^2}{2} + 6x + 12 \ln |x| - \frac{8}{x} + c \]

(c) \[ \frac{x^2}{2} - 6x + 12 \ln |x| - \frac{8}{x} + c \]

(d) \[ \frac{x^2}{2} - 6x + 6 \ln |x| - \frac{4}{x} + c \]

(e) \[ \frac{x^2}{2} + 6x - 12 \ln |x| + \frac{8}{x} + c \]
5. If \( F(x) = \int_1^x f(t) \, dt \), where \( f(t) = \int_1^{t^2} \frac{\sqrt{1 + u^4}}{u} \, du \), then \( F''(2) = \)

(a) \( \sqrt{257} \)
(b) \( \sqrt{255} \)
(c) \( \sqrt{253} \)
(d) \( \sqrt{259} \)
(e) \( \sqrt{261} \)

6. The volume of the solid resulting from the region: \( y = -x^2 + 6x - 8 \); \( y = 0 \) which has been rotated about the \( y \)-axis is given by the definite integral:

(a) \( \int_2^4 2\pi \, x \left[-x^2 + 6x - 8\right] \, dx \)
(b) \( \int_2^4 \pi \, x \left[-x^2 + 6x - 8\right] \, dx \)
(c) \( \int_0^8 2\pi \, x \left[-x^2 + 6x - 8\right] \, dx \)
(d) \( \int_2^4 2\pi \left[-x^2 + 6x - 8\right] \, dx \)
(e) \( \int_0^4 2\pi \, x \left[-x^2 + 6x - 8\right] \, dx \)
7. If \( f \) is an even function such that \( \int_{-1}^{1} f(t) \, dt = 5 \) and \( \int_{-2}^{2} f(t) \, dt = 2 \),
then \( \int_{1}^{2} f(t) \, dt = \\
(a) \ -\frac{3}{2} \\
(b) \ \frac{3}{2} \\
(c) \ 3 \\
(d) \ -3 \\
(e) \ 0 \\

8. \( \int_{-3}^{0} (|x - 1| + \sqrt{9 - x^2}) \, dx = \\
\text{(Hint: You may interpret the integral as an area)} \\
(a) \ \frac{9\pi + 30}{4} \\
(b) \ \frac{9\pi + 26}{4} \\
(c) \ \frac{9\pi + 34}{4} \\
(d) \ \frac{7\pi + 30}{4} \\
(e) \ \frac{7\pi + 34}{4} \)
9. \[ \lim_{{n \to \infty}} \frac{2}{n} \sum_{{i=1}}^{n} \frac{1}{1 + \left(\frac{i-1}{n}\right)^2} = \]

(a) \( \frac{\pi}{2} \)
(b) \( \frac{\pi}{4} \)
(c) 0
(d) 1
(e) 2

10. \[ \int \sin^2 x \, dx = \]

(a) \( \frac{x}{2} - \frac{\sin 2x}{4} + c \)
(b) \( -\cos x + c \)
(c) \( \frac{1}{2} \cos^2 x + c \)
(d) \( \frac{1}{2} \cos 2x + c \)
(e) \( \frac{x}{2} - \cos x + c \)
11. If the region enclosed by the curves \( y = x \) and \( y = x^3 \), where \( x \geq 0 \), is rotated about the \( x \)-axis, then the volume of the solid obtained is equal to

(a) \( \frac{4\pi}{21} \)
(b) \( \frac{\pi}{4} \)
(c) \( \frac{11\pi}{21} \)
(d) \( \frac{7\pi}{21} \)
(e) \( \frac{\pi}{7} \)

12. The area of the region bounded by the curves \( y = \sin x \), \( y = \cos x \), \( x = 0 \) and \( x = \frac{\pi}{2} \) is equal to

(a) \( 2\sqrt{2} - 2 \)
(b) \( 4\sqrt{2} + 2 \)
(c) \( 2\sqrt{2} + 2 \)
(d) \( 4 \)
(e) \( \sqrt{2} - 1 \)
13. The area of the region enclosed by the curves, \( y = x^2 - 4 \), \( y = -2x + 4 \), and \( y = -4 \) is equal to

(a) \( \frac{20}{3} \)

(b) \( \frac{17}{3} \)

(c) \( \frac{8}{5} \)

(d) \( \frac{12}{5} \)

(e) 3

14. A particle moves along a line so that its velocity at time \( t \) is \( v(t) = t - t^2 \). The distance traveled by the particle during the time period \( 0 \leq t \leq 2 \) is:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5
15. The region whose area is equal to \( \lim_{n \to \infty} \frac{4}{3} \left( \frac{\pi + 3}{n} \right) \sum_{i=1}^{n} \sin \left( \frac{\pi i + 3i - 3n}{3n} \right)^2 \)
is the region

(a) under the graph of \( y = 4 \sin x^2 \) from \(-1\) to \(\frac{\pi}{3}\).

(b) under the graph of \( y = \sin x^2 \) from \(1\) to \(\frac{\pi}{3}\).

(c) under the graph of \( y = \sin \left( \frac{x^2}{4} \right) \) from \(-1\) to \(\frac{\pi}{4}\).

(d) under the graph of \( y = 4 \sin x^2 \) from \(1\) to \(\frac{\pi}{4}\).

(e) under the graph of \( y = \frac{4}{3} \sin x^2 \) from \(3\) to \(\pi\).

16. The slope of the line tangent to the curve \( g(x) = \int_{0}^{x^3} \sqrt{t + e^t} \, dt \)
at \( x = 2 \) is

(a) \( 12\sqrt{8} + e^8 \)

(b) \( 8\sqrt{8} + e^8 \)

(c) \( 8\sqrt{2} + e^2 \)

(d) \( 12\sqrt{2} + e^2 \)

(e) \( 12\sqrt{8} + e^2 \)
17. \[ \int_{0}^{\pi/3} \sin x \cos 2x \, dx = \]

(a) \( \frac{1}{12} \)

(b) \( \frac{1}{2} \)

(c) \( \frac{1}{3} \)

(d) \( \frac{1}{4} \)

(e) \( \frac{1}{6} \)

18. Using cylindrical shells, the volume of the solid that is generated when the region enclosed by \( y = x^3, y = 1, x = 0 \) is revolved about \( y = 1 \), is

(a) \( \frac{9\pi}{14} \)

(b) \( \frac{7\pi}{15} \)

(c) \( \frac{15\pi}{21} \)

(d) \( \frac{3\pi}{14} \)

(e) \( \frac{17\pi}{14} \)
19. The volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the $x-$ axis are squares is

(a) $\frac{1}{30}$

(b) $\frac{1}{12}$

(c) $\frac{1}{18}$

(d) $\frac{1}{36}$

(e) $\frac{1}{24}$

20. Let $m$ and $M$ be the absolute minimum and the absolute maximum values respectively, of an integrable function $f$ over a closed interval $[3, 5]$. If an estimation, based on $m$ and $M$, of the integral $\int_3^5 f(x) \, dx$ lies in the interval $[A, B]$, then $A + B =$

(a) $2(M + m)$

(b) $2(M - m)$

(c) $8(M + m)$

(d) $8(M - m)$

(e) $2Mm$