Serial No.:______ Student Name:_______________________ Student Number: ______
Instructor: M. Z. Abu-Sbeih Math 102- Q5 Date: 24-5-2010

<u>Problem 1:</u> (15 points) Determine if the series is convergent or divergent.

(a) $\sum \frac{\arctan(n)}{n^2+1}$

Solution: $a_n = \arctan(n)/(1+n^2)$. Let $f(x) = \arctan x/(1+x^2)$. Then, f is positive and continuous on $[1,\infty)$ and since $f'(x) = \frac{1-2x \arctan x}{(1+x^2)^2} < 0$ for all x in $[1,\infty)$ we conclude that f is also decreasing on this interval.

$$\int_1^\infty f(x)dx = \lim_{t\to\infty}\int_1^t \frac{\arctan x}{1+x^2}dx$$
$$= \lim_{t\to\infty}\frac{1}{2}\left[\left(\arctan t\right)^2 - \left(\arctan 1\right)^2\right].$$
$$= \frac{1}{2}\left(\frac{\pi^2}{4} - \frac{\pi^2}{16}\right) = \frac{3\pi^2}{32}$$

Thus, by the integral test the series $\sum \frac{\arctan(n)}{1+n^2}$ is convergent.

(b) $\sum \frac{n}{(4+n^2)^{\frac{3}{4}}}$

Solution: $a_n = \frac{n}{(4+n^2)^{3/4}}$. Let $b_n = \frac{1}{\sqrt{n}}$. Then , $\lim_{n\to\infty} a_n/b_n = \lim_{n\to\infty}\frac{n\sqrt{n}}{\left(4+n^2\right)^{3/4}} = 1$, and $\sum b_n$ diverges (p-series with $p = \frac{1}{2}$) and therefore by the LCT (the limit comparison test) $\sum a_n$ diverges . (Recall that if $0 < \lim_{n\to\infty} a_n/b_n < \infty$ then $\sum a_n$ diverges iff $\sum b_n$ diverges , or, $\sum a_n$ converges iff $\sum b_n$ converges .) Can you apply the B.C.T.(the basic comparison test)?

(c) $\sum \frac{4^n(n!)}{(2n)!}$

Solution: $a_n = \frac{4^n n!}{(2n)!} \Rightarrow \frac{a_n+1}{a_n} = \frac{4^{n+1}(n+1)!}{(2(n+1))!}\frac{(2n)!}{4^n n!} = \frac{2}{2n+1}$. Thus $\lim_{n\to\infty}\frac{a_n+1}{a_n} = 0 = L < 1$, and hence by the ratio test the series is convergent. (Recall that if $L = \lim_{n\to\infty}\frac{a_n+1}{a_n}$ or $L = \lim_n \sqrt[n]{a_n}$ then the positive term-series $\sum a_n$ converges if $L < 1$, diverges if $L > 1$ and the test is inconclusive if $L = 1$).

Problem 2: **(10 points)** determine if the series is absolutely convergent, conditionally convergent or divergent.

(d) $\sum (-1)^{n-1} \frac{2.4.6\cdots(2n)}{n!}$

Solution: Alternating series with $a_n = \frac{2 \cdot 4 \cdot 6 \cdots 2n}{n!} = \frac{2^n \cdot n!}{n!} = 2^n \xrightarrow{n\to\infty} \infty$. Hence the series $\sum (-1)^{n-1} a_n$ diverges (since its general term $(-1)^{n-1} a_n \not\to 0$ as $n \to \infty$).

(e) $\sum \frac{(-3)^{n+1}}{2^{2n}}$

Solution: The alternating series $\sum \frac{(-3)^{n+1}}{2^{2n}} = \sum (-1)^{n+1} 3 \left(\frac{3}{4}\right)^n$ converges absolutely since $\sum 3\left(\frac{3}{4}\right)^n = 3\sum \left(\frac{3}{4}\right)^n$ converges (geometric series with $r = 3/4 < 1$).