King Fahd University of Petroleum and Minerals
Math & Stat. Departement
Quiz (4)

Name	ID	SEC	Sr

Q1) Test the following series for convergence (justify your answer)

i. $\sum_{n=1}^{\infty} \frac{n3^n}{5^n}$

$\sqrt[n]{\frac{n3^n}{5^n}} = n^{1/n}\frac{3}{5}$, $\lim_{n\to\infty} n^{1/n} = 1$, so,

$\lim_{n\to\infty} \sqrt[n]{\frac{n3^n}{5^n}} = \frac{3}{5} < 1$,

$\therefore$ by Root test, $\sum_{n=1}^{\infty} \frac{n3^n}{5^n}$ is absolutly convergent

You can use the Ratio test.
$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\cdot\frac{3}{5}\right| = 3/5 < 1$

ii. $1+\frac{1}{2^2\sqrt{2}}+\frac{1}{3^2\sqrt{3}}+\frac{1}{4^2\sqrt{4}}+\cdots = \sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$

$= \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ p-series, $p = \frac{5}{2} > 1$,

so it is convergent

iii. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n}{n+1}\right)^n$. Let $b_n = \left(\frac{n}{n+1}\right)^n$.

$b_n = \left(\frac{n+1}{n}\right)^{-n} = \left(1+\frac{1}{n}\right)^{-n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$, $\lim_{n\to\infty} b_n = \frac{1}{e} \neq 0$

so $\lim_{n\to\infty} (-1)^n b_n$ does not exist, $\therefore$, by divergence test, $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n}{n+1}\right)^n$ is divergent

Q2) Find the smallest number of terms we need to add such that **the error** of the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is less than 0.1 , $b_n = \frac{1}{\sqrt{n}}$, b_n ~~↓~~ $\lim_{n\to\infty} b_n = 0$

so, $|s - S_n| \le b_{n+1}$ 1

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$b_{n+1} < 0.1 \Rightarrow \frac{1}{\sqrt{n+1}} < 0.1 \Rightarrow \sqrt{n+1} > 10 \Rightarrow n+1 > 100$

1 $\Rightarrow \boxed{n \ge 99}$.

$\therefore$, the smallest number of n is 100. 1

Q3) Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{4^n n^3}$ (justify your answer)

$\frac{a_{n+1}}{a_n} = \frac{x^{n+1}}{4^{n+1}(n+1)^3} \cdot \frac{4^n \cdot n^3}{x^n} = \frac{x}{4}\left(\frac{n}{n+1}\right)^3$ 1

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|}{4} < 1 \Rightarrow |x| < 4$, so $\boxed{R = 4}$

1 1

$\Downarrow$

$-4 < x < 4$.

At $x = 4$, $\sum_{n=1}^{\infty} \frac{4^n}{4^n n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ 1, p-series, $p = 3 > 0$, so convergent

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At $x = -4$, $\sum_{n=1}^{\infty} \frac{(-4)^n}{4^n n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ convergent by alternating test.

$\therefore$ $[-4, 4]$ is the interval of convergence

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