

MATH 102 - Quiz 1

Section number:

Student ID:

Instructions: You are required to attempt all questions. Each is worth 5 points.

1. Determine a region whose area is equal to the given limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(5 + \frac{2i}{n}\right)^{10}$$

Either Solution 1:

Width of interval is $\Delta x = \frac{2}{n}$. So the gap is 2. $x_i = 5 + (\Delta x)i \Rightarrow x \in [5, 7]$.
 $f(x) = x^{10}$

or Solution 2:

Width of interval is $\Delta x = \frac{2}{n}$. So the gap is 2. $x_i = (\Delta x)i \Rightarrow x \in [0, 2]$.
 $f(x) = (x + 5)^{10}$

2. Find the derivative of the function
- $g(x) = \int_{\tan(x)}^{x^2} \frac{1}{\sqrt{2+t^4}}$

Solution:

To do this problem, we need to apply the chain rule in addition to the Fundamental Theorem of Calculus. Suppose $f(x) = \frac{1}{\sqrt{2+x^4}}$.

If $F'(x) = f(x)$ and let $\alpha(x) = \tan(x)$ and $\beta(x) = x^2$, then $g(x) = F(\beta(x)) - F(\alpha(x))$.

$$g'(x) = F'(\beta(x)) \times \beta'(x) - F'(\alpha(x)) \times \alpha'(x)$$

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$$g'(x) = \frac{2x}{\sqrt{2+x^8}} - \frac{\sec^2(x)}{\sqrt{2+\tan^4(x)}}$$

3. If
- $f(x)$
- is continuous and
- $\int_0^9 f(x)dx = 4$
- , then find
- $\int_0^3 xf(x^2)dx$

Solution:

let $u = x^2$, then $\frac{du}{2} = xdx$.

$$\int_0^3 xf(x^2)dx = \frac{1}{2} \int_{0^2}^{3^2} f(u)du = \frac{4}{2} = 2.$$