MATH 102 - Quiz 2 Section number: Student ID:

Instructions: You are required to attempt all questions. Write your steps clearly. Failing to do so will result in an automatic zero for the question.

1. Find the area of the region enclosed by the curves $y = |x|$ and $y = x^2 - 2$.

Solution:
When $x < 0$, $y = -x$ and $y = x$ when $x > 0$.
First intersection point in $x < 0$: $-x = x^2 - 2 \rightarrow x = -2, 1$. So the relevant point is at $x = -2$
Second intersection point in $x > 0$: $x = x^2 - 2 \rightarrow x = 2, -1$. So the relevant point is at $x = 2$.
$A = \displaystyle\int_{-2}^{0} [-x - (x^2 - 2)]dx + \int_{0}^{2} [x - (x^2 - 2)]dx$
$A = [-\frac{x^2}{2} - \frac{x^3}{3} + 2x]_{-2}^{0} + [\frac{x^2}{2} - \frac{x^3}{3} + 2x]_{0}^{2}$
$A = -[-\frac{(-2)^2}{2} - \frac{(-2)^3}{3} + 2(-2)] + [\frac{2^2}{2} - \frac{2^3}{3} + 2(2)] = \frac{20}{3}$.

2. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$ and $y = 8$ about the x-axis. Sketch the region and the axis of rotation as part of your solution. No marks will be given for the entire question if the region and the axis of rotation are not drawn.

Solution:
Sketch the region of interest and axis of rotation first.
Shell method:
axis of rotation is y-axis
$r = y,\ f = y^{1/3}$ $V = \displaystyle\int_{0}^{8} 2\pi y(y^{1/3})dy = \int_{0}^{8} 2\pi y^{4/3}dy$
$V = 2\pi\frac{3}{7}(8)^{7/3} = \frac{768\pi}{7}$

3. Find the volume of the torus generated by revolving a circle $x^2 + y^2 = 4$ about the line $x = 3$. Sketch the region being rotated as part of the solution. No marks will be given for the entire question if the region and axis of rotation are not drawn.

Solution:
Sketch the region of interest and axis of rotation first.
Shell method: $V = \displaystyle\int_{-2}^{2} 2\pi(3 - x) \cdot 2\sqrt{4 - x^2}dx$
$V = 12\pi \displaystyle\int_{-2}^{2} \sqrt{4 - x^2}dx - 4\pi \int_{-2}^{2} x\sqrt{4 - x^2}dx = 24\pi^2$
(recall Chapter 5, first integral is area of upper semi circle whereas the second integral is odd function so is zero.)