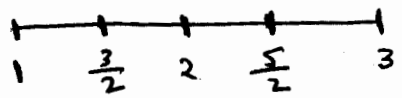
King Fahd University of Petroleum and Minerals
Math & stat. Departement
Quiz (1)

Name	KEY	ID	SEC	Sr

Q1) Find the Riemann sum for $f(x)=\frac{15}{x}$, $1\le x\le 3$ with four subintervals and right endpoints as sample points.

$n=4$, $\Delta x=\frac{b-a}{n}=\frac{3-1}{4}=\frac{1}{2}$



$$R_4=\frac{1}{2}\left[f\left(\frac{3}{2}\right)+f(2)+f\left(\frac{5}{2}\right)+f(3)\right]$$

$$=\frac{1}{2}\left[10+\frac{15}{2}+6+5\right]=\frac{1}{2}\left[\frac{57}{2}\right]=\frac{57}{4}$$

Q2) Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4i}{n^2}+\frac{3}{n}\right)$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4i}{n^2}+\frac{3}{n}\right)=\lim_{n\to\infty}\frac{4}{n^2}\sum_{i=1}^{n}i+\lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}1$$

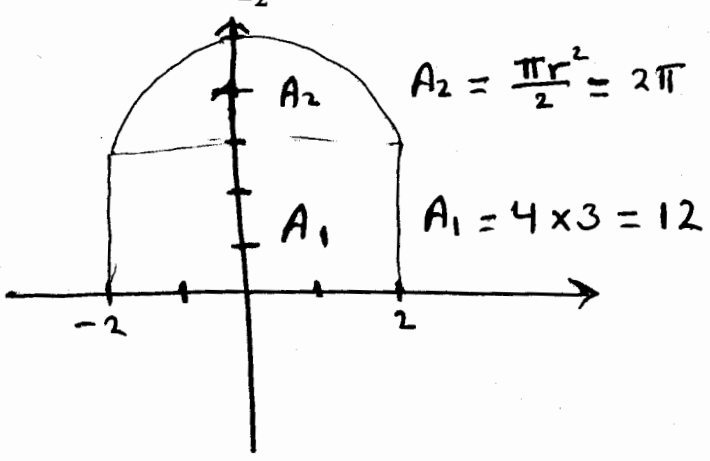
$$\lim_{n\to\infty}\frac{4(n)(n+1)}{2n^2}+\lim_{n\to\infty}\frac{3n}{n}$$

$$=2+3=5$$

Q3) By interpreting the integral as an area, find the value of $\int_{-2}^{2}\left(3+\sqrt{4-x^2}\right)dx$

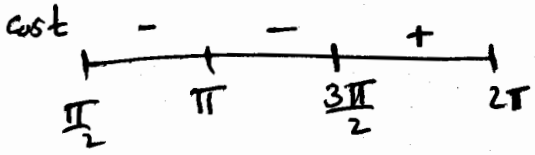
$$\int_{-2}^{2}\left(3+\sqrt{4-x^2}\right)dx$$

$$=A_1+A_2=12+2\pi$$



Q4) A particle moves along a line so that its velocity at time t is $v(t) = \cos t$ (in meter per second). Find the total distance traveled by the partical during the time period $\frac{\pi}{2} \le t \le 2\pi$.

$$\text{distance} = \int_{\frac{\pi}{2}}^{2\pi} |\cos t|\, dt$$



$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\cos t\, dt + \int_{\frac{3\pi}{2}}^{2\pi} \cos t\, dt = -\sin t \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin t \Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= (1+1) + (0+1) = 3$$

Q5) Evaluate the indefinite integral $\int x^3 \sqrt{x^2+1}\, dx$

let $u = x^2+1 \Rightarrow du = 2x\, dx$ and $x^2 = u-1$

$$\int x^3 \sqrt{x^2+1}\, dx = \frac{1}{2}\int (2x)\, x^2 \sqrt{x^2+1}\, dx$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\, du = \frac{1}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$

$$= \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right] + C = \frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}} + C$$

Q6) If $G(x) = \int_{\sin x}^{\cos(3x)} \frac{1}{\sqrt{1+4t^2}}\, dt$, then find $G'\left(\frac{\pi}{2}\right)$

$$G'(x) = \frac{1}{\sqrt{1+4(\cos^2 3x)}} \cdot (-3\sin 3x) - \frac{1}{\sqrt{1+4\sin^2 x}} \cdot \cos x$$

$$G'\left(\frac{\pi}{2}\right) = 3 - 0$$

$$= 3$$