Quiz #5

Question (10 points total)

(a) If

$$y = \frac{\cos x}{1 + \sin x},$$

then find dy/dx;

(b) Evaluate the integral

$$\int x \sin \frac{x}{2}\, dx.$$

Solution:

(a) You need to use the product rule:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{(1+\sin x)(\frac{d}{dx}\cos x) - (\cos x)\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2} \\
&= \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2} \\
&= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} \\
&= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1}{1+\sin x}
\end{aligned}$$

(b) Here we will use integration by parts. Let $u = x$ and $dv = \sin \frac{x}{2}$, then $du = dx$ and $v = -2\cos \frac{x}{2}$. So,

$$\begin{aligned}
\int x \sin \frac{x}{2}\, dx &= -2x \cos \frac{x}{2} - \int \left(-2 \cos \frac{x}{2}\right)\, dx \\
&= -2x \cos \frac{x}{2} + 2 \int \cos \frac{x}{2}\, dx \\
&= -2x \cos \frac{x}{2} + 2(2 \sin \frac{x}{2}) + C \\
&= -2x \cos \frac{x}{2} + 4 \sin \frac{x}{2} + C.
\end{aligned}$$

Note: Points will be deducted for incomplete or incorrect answers. Points will also be deducted for not fully or properly showing your work.