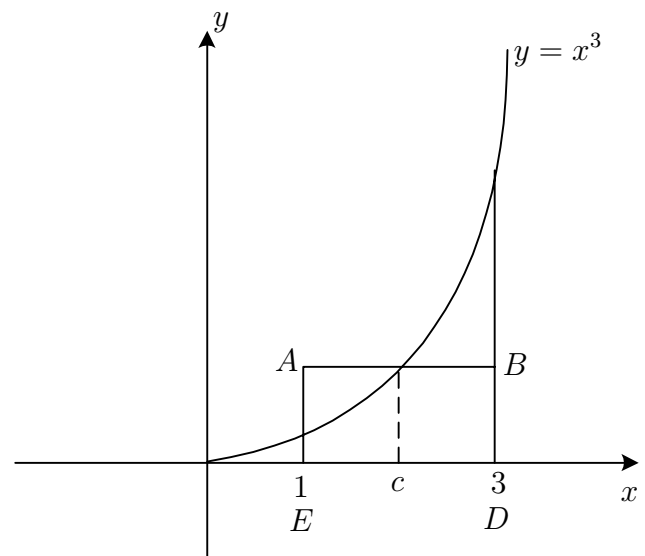


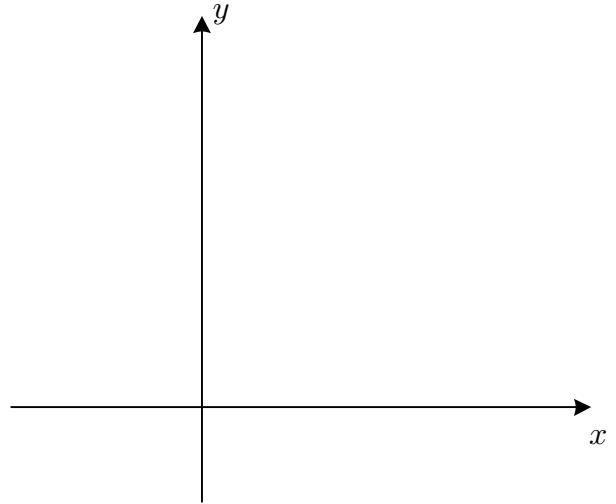
1. (5-points) Find a number  $c$ ,  $1 < c < 3$ , such that the area of the rectangle  $ABDE$  in the adjacent figure is equal to the area of the region bounded above by  $y = x^3$ , and bounded below by  $y = 0$  from  $x = 1$  to  $x = 3$ .



2. (5-points) List the first five terms of the sequence  $\{a_n\}$  where  $a_1 = 1$  and  $a_{n+1} = \frac{1}{1 + a_n}$ .

3. (5-points) Find a formula for the general term  $a_n$  of the sequence  $\left\{ \frac{2^2}{2 \cdot 3}, \frac{4^2}{3 \cdot 5}, \frac{6^2}{4 \cdot 7}, \frac{8^2}{5 \cdot 9}, \dots \right\}$ . Then determine whether the sequence converges or diverges.

4. (12-points) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the graphs of  $y = e^{-x}$ ,  $x = 0$ ,  $y = 0$ , and  $x = 1$  about the line  $x = -2$ .  
[Sketch the region and a typical rectangle]



5. (4-points) Write out the form of the partial fraction decomposition of  $\frac{x+5}{x^2(x^2-x-2)(x^2+5)^2}$ . Do not determine the numerical values of the coefficients.

6. Evaluate the following integrals:

(a) (6-points)  $\int \frac{\sqrt{\csc \theta}}{\sec^3 \theta} d\theta.$

(b) (6-points)  $\int \frac{\sqrt{x}}{1 + \sqrt{x}} dx.$

(c) (8-points)  $\int x \tan^{-1} \left( \frac{1}{x} \right) dx.$

(d) (8-points)  $\int \frac{x^2}{(4 - 9x^2)^{3/2}} dx.$

(e) (12-points)  $\int \frac{7x^2 + x}{(3x - 1)(x^2 + 1)} dx.$

7. (8-points) Show that the integral test is applicable to the series

$\sum_{n=0}^{\infty} \frac{1}{n^2 + 4n + 5}$ , then use it to determine whether the series converges or diverges.

8. (5-points) Evaluate  $\int_0^4 \frac{1}{(x-1)^2} dx$  if possible.

9. (6-points) The  $n$ -th partial sum  $S_n$  of the series  $\sum_{n=1}^{\infty} a_n$  is given by

$$S_n = \frac{n-1}{n+1}. \quad \text{Find } a_n \text{ and the sum of the series.}$$

10. Determine whether each of the following series is convergent or divergent. Find the sum of the series if it is convergent.

(a) (5-points)  $\sum_{n=1}^{\infty} \left( \frac{4\sqrt{n^3} + 5\sqrt[3]{n}}{n^2} \right).$

(b) (5-points)  $\sum_{n=1}^{\infty} \frac{(-2)^{n-2}}{5^{n-1}}.$