King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Summer (093)
Tuesday, August 24, 2010

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5 per question

This exam was prepared using mcqs
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Math 102
Final Exam
Summer (093)
Tuesday, August 24, 2010
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\displaystyle\int_0^1 \frac{1}{\sqrt{x+2}-\sqrt{x+1}}\,dt =$

(a) $\frac{2}{3}(3\sqrt{3}-1)$

(b) $2\sqrt{3}$

(c) $\frac{2}{3}(\sqrt{3}+1)$

(d) $\frac{4}{3}\sqrt{3}$

(e) $\frac{2}{3}(3\sqrt{3}+2)$

2. The volume of the solid generated by rotating the region bounded by the curve $xy = 1$, and the lines $y = 0$, $x = 1$ and $x = 3$ about the y-axis, is equal to

(a) 4π

(b) 2π

(c) $\frac{\pi}{2}$

(d) $\frac{3\pi}{4}$

(e) $\frac{\pi}{4}$

3. The sum of the series $1 - \ln 3 + \dfrac{(\ln 3)^2}{2!} - \dfrac{(\ln 3)^3}{3!} + \cdots$ is

(a) $\dfrac{1}{3}$

(b) $\dfrac{1}{1+\ln 3}$

(c) -3

(d) $\dfrac{1}{1-\ln 3}$

(e) $-\dfrac{1}{3}$

4. If $\dfrac{x^5+x^3+1}{x^3+x}$ is written in the form $P(x) + \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$ where $P(x)$ is a polynomial and A, B and C are constants, then $P(2) + A + B + C =$

(a) 4

(b) 3

(c) -2

(d) 6

(e) 2

5. If the substitution $t = \tan\left(\frac{x}{2}\right)$ is used, then the integral $\int_0^{\pi/2} \frac{dx}{3\sin x + 4\cos x + 8}$, transforms to

(a) $\int_0^1 \frac{1}{2t^2 + 3t + 6}\, dt$

(b) $\int_0^{\pi/2} \frac{1}{2t^2 + 7t + 4}\, dt$

(c) $\int_0^1 \frac{1}{2t^2 + 7t + 6}\, dt$

(d) $\int_0^{\pi/4} \frac{1}{2t^2 + 5t + 6}\, dt$

(e) $\int_0^1 \frac{1}{2t^2 + 3t + 10}\, dt$

6. The sum of the series $\sum_{n=1}^{\infty} [\tan^{-1}(n+1) - \tan^{-1} n]$ is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$

(d) 0

(e) 1

7. $\displaystyle\int_0^{\pi/3} \frac{\tan\theta}{\sec\theta + 1} d\theta =$

(a) $\ln\left(\frac{4}{3}\right)$

(b) $\ln(4\sqrt{3})$

(c) $\ln 4$

(d) $\ln 3$

(e) $\ln\sqrt{3}$

8. The alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n^3}}$ is

(a) conditionally convergent

(b) divergent by the alternating series test

(c) divergent by the n-th root test

(d) absolutely convergent

(e) convergent by the integral test

9. The sequence $\{\sqrt{2}(\sqrt{12}-\sqrt{2}),\ \sqrt{3}(\sqrt{13}-\sqrt{3}),\ \sqrt{4}(\sqrt{14}-\sqrt{4}),\ldots\}$ converges to

(a) 5

(b) 0

(c) 1

(d) 4

(e) $\frac{1}{2}$

10. The series $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n+5^n}$ is

(a) Absolutely convergent

(b) Divergent by the integral test

(c) A convergent p-series

(d) A divergent geometric series

(e) Conditionally convergent

11. If f is a differentiable function such that $f(2) = -5$ and $f(6) = 3$, then $\int_1^4 \left(\frac{2\sqrt{x}+1}{\sqrt{x}}\right) f'(x+\sqrt{x})dx =$

(a) 16

(b) 4

(c) -4

(d) 8

(e) 15

12. The power series representation of $\frac{1}{3+x^2}$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} x^{2n}$

(b) $\sum_{n=0}^{\infty} \frac{1}{3^n} x^{2n}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} x^{2n+2}$

(d) $\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^{2n}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n} x^{2n}$

13. $\displaystyle\int_{-3/2}^{-1/2} \frac{|2x+2|}{x^2+2x} dx =$

(a) $\ln \frac{9}{16}$

(b) $\ln \frac{3}{4}$

(c) $\ln \frac{2}{3}$

(d) $\ln \frac{15}{16}$

(e) 0

14. If the integral $\displaystyle\int \frac{\sin x - x}{x} dx$ is evaluated as an infinite series, then the coefficient of x^5 is equal to

(a) $\frac{1}{600}$

(b) $-\frac{1}{300}$

(c) $\frac{1}{450}$

(d) $-\frac{1}{450}$

(e) $\frac{1}{900}$

15. The Taylor series of $f(x) = 2^{x-1}$ at $x = 1$ is

(a) $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!}(x-1)^n$

(b) $\sum_{n=0}^{\infty} \left(\frac{\ln 2}{2}\right)^n \frac{(x-1)^n}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{(2\ln 2)^n}{n!}(x-1)^n$

(d) $\sum_{n=0}^{\infty} \frac{(\ln 2)^{2n}}{n!}(x-1)^n$

(e) $\sum_{n=0}^{\infty} (-1)^n (\ln 2)^n \frac{(x-1)^n}{n!}$

16. The volume of the solid generated by rotating the region bounded by the curves $y = x^3$, $y = x - 1$, $x = 1$ and $x = 2$, about $y = 8$ is given by the integral

(a) $\int_1^2 \pi(17 - 18x + x^2 + 16x^3 - x^6)dx$

(b) $\int_1^8 \pi(18 + 14x - x^2 - 16x^3 - x^6)dx$

(c) $\int_1^2 \pi(17 + 12x + x^2 + 16x^3 - x^6)dx$

(d) $\int_1^8 \pi(18 + 14x - x^2 - 16x^3 - x^6)dx$

(e) $\int_1^2 \pi(17 - 19x + x^2 - 18x^3 - x^6)dx$

17. The area of the region bounded by the x-axis and the curve $y = (x-1)^3$ from $x = 0$ to $x = 2$ is equal to

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 0

(e) $\frac{3}{4}$

18. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-5)^n x^n}{\sqrt{n+4}}$ is

(a) $\left(-\frac{1}{5}, \frac{1}{5}\right]$

(b) $\left(-\frac{5}{2}, \frac{5}{2}\right]$

(c) $\left(-\frac{1}{5}, \frac{1}{5}\right)$

(d) $\left(-\frac{5}{2}, \frac{5}{2}\right)$

(e) $\left[-\frac{1}{5}, \frac{1}{5}\right)$

19. The average value of the function $f(x) = \dfrac{1 - \tan^2 x}{\sec^2 x}$ over the interval $\left[0, \dfrac{\pi}{12}\right]$ is

(a) $\dfrac{3}{\pi}$

(b) $\dfrac{4}{\pi}$

(c) $\dfrac{8}{\pi}$

(d) $\dfrac{1}{\pi}$

(e) $\dfrac{2}{\pi}$

20. The series $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n} + \ln n}{n^2 + 1}$

(a) converges by the limit comparison test with $b_n = \dfrac{1}{n^{3/2}}$

(b) diverges by the limit comparison test with $b_n = \dfrac{1}{\sqrt{n}}$

(c) diverges by the integral test

(d) diverges by the comparison test with $b_n = \dfrac{1}{\sqrt{n^2 + 1}}$

(e) diverges by the comparison test with $b_n = \dfrac{1}{\ln n}$

21. $\int \sqrt{4x - x^2}\, dx =$

(a) $2\sin^{-1}\left(\dfrac{x-2}{2}\right) + \dfrac{1}{2}(x-2)\sqrt{4x-x^2} + C$

(b) $\dfrac{1}{4}\sin^{-1}\left(\dfrac{x-2}{2}\right) + \dfrac{3}{2}(x-2)\sqrt{4x-x^2} + C$

(c) $(x-2)\sqrt{4x-x^2}\sin^{-1}\left(\dfrac{x-2}{2}\right) + C$

(d) $2\sin^{-1}\left(\dfrac{x-2}{2}\right) + \dfrac{3}{2}(x-2)\sqrt{4x-x^2} + C$

(e) $\sin^{-1}(x-2) + (x-2)\sqrt{4x-x^2} + C$

22. The area of the surface generated by rotating the curve $y = \int_1^x \sqrt{t-1}\, dt, \quad 1 \le x \le 4,$ about the y-axis is

(a) $\dfrac{124\pi}{5}$

(b) $\dfrac{81\pi}{5}$

(c) $\dfrac{139\pi}{5}$

(d) 44π

(e) 32π

23. $\displaystyle\int \frac{e^{2x}}{e^{4x}+4e^{2x}+20}dx =$

(a) $\frac{1}{8}\tan^{-1}\left(\frac{e^{2x}+2}{4}\right)+C$

(b) $\frac{1}{2}\ln\left|\frac{e^{2x}+1}{e^{2x}-1}\right|+C$

(c) $\frac{1}{6}\tan^{-1}(e^{2x}+2)+C$

(d) $\frac{1}{8}\ln\left|\frac{e^{2x}-1}{e^{2x}+1}\right|+C$

(e) $\frac{1}{24}\tan^{-1}\left(\frac{e^{2x}+4}{2}\right)+C$

24. The length of the curve $y=\ln(\sec x),\quad 0\le x\le \frac{\pi}{4}$, is

(a) $\ln(\sqrt{2}+1)$

(b) $\ln\frac{\sqrt{2}}{3}$

(c) $\ln(2\sqrt{2}-1)$

(d) $\ln 2\sqrt{3}$

(e) $\ln(\sqrt{2}+\sqrt{3})$

25. $\int x^7 e^{x^4} dx =$

(a) $\frac{1}{4}e^{x^4}(x^4 - 1) + C$

(b) $\frac{1}{16}e^{x^4}(x^4 + 8) + C$

(c) $\frac{1}{4}e^{x^4}(x^4 - 8) + C$

(d) $\frac{7}{4}e^{x^4}(x^4 - 1) + C$

(e) $\frac{1}{4}e^{x^4}(x^4 - 7) + C$

26. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n+5)!\, x^n}{3 \cdot 5 \cdot 7 \cdots\ (2n+1)}$ is

(a) 2

(b) ∞

(c) 1

(d) 5

(e) $\frac{5}{2}$

27. The smallest number of terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/2}}$ that we need to add so that $|\text{error}| < 0.008$ is

(a) 25

(b) 15

(c) 35

(d) 45

(e) 30

28. The improper integral $\int_0^e \ln x \, dx$

(a) converges to 0

(b) is divergent

(c) converges to $-e$

(d) converges to e

(e) converges to 1