MATH 202.2 (092)

Quiz 2 (Chap. 2.4-4.1.2) Duration: 15mn

Name: ID number:

1.) Find a suitable substitution that transforms the DE $8xy^2dx + (12x^2y + 40y^2 - 2)dy = 0$ into a separable DE. Find the new separable equation and solve it. Find the solution y which satisfies $y(1) = 1$.

Solution

Let $M(x,y) = 8xy^2$

and $N(x,y) = 12x^2y + 40y^2 - 2$.

$M_y = 16xy$

$N_x = 24xy$

$M_y \neq N_x \Rightarrow$ the DE is not exact

We evaluate $\frac{N_x - M_y}{M}$

$$\frac{N_x - M_y}{M} = \frac{8xy}{8xy^2} = \frac{1}{y}$$

An integrating factor is

$$\mu(y) = e^{\int \frac{dy}{y}} = e^{\ln|y|} = y, \quad y > 0$$

Multiplying the DE by y, we find.

$$8xy^3dx + (12x^2y^2 + 40y^3 - 2y)\,dy = 0$$

The new $M(x,y) = 8xy^3$

and new $N(x,y) = 12x^2y^3 + 40y^3 - 2y$

And we have

$$M_y = 24xy^2 = N_x$$

The new DE is now exact.

$\frac{\partial f}{\partial x} = 8y^3x$ and

$\frac{\partial f}{\partial y} = 12x^2y^2 + 40y^3 - 2y$

From the 1st ~~rel~~ relation, we get

$$f(x,y) = 4y^3x^2 + g(y)$$

$$\frac{\partial f}{\partial y}(x,y) = 12y^2x^2 + g'(y)$$

This shows that

$$g'(y) = 40y^3 - 2y$$

and $g(y) = 10y^4 - y^2$

So that, the solution of DE is given by

$$\boxed{4y^3x^2 + 10y^4 - y^2 = C}$$

Now, $y(1) = 1 \Rightarrow$

$4 + 10 - 1 = C \Rightarrow C = 13$

Thus, $\boxed{4y^3x^2 + 10y^4 - y^2 = 13}$