MATH 202.7 (092)

Quiz 2 (Chap. 2.4-4.1.2) Duration: 20mn

Name: ID number:

1.)(6pts) Find a suitable substitution that transforms the DE $(\sin x - y^2 \cos x)dx + \frac{1}{y}dy = 0$ into a linear DE. Find the new linear equation, but do not solve it.

2.)(4pts) Solve the DE $\frac{dy}{dx} - 2\sin x y = -2\sin x$ subject to $y(0) = 1$.

Solution

1-) We rewrite the DE in its standard form

$$\frac{dy}{dx} + \sin x\, y = y^3 \cos x$$

This is a Bernoulli equation. To transform it into a linear equation, we may set $u = y^{-2}$

$$\frac{du}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$= -2u^{3/2}\frac{dy}{dx}$$

Substituting into the DE we find

$$-\frac{1}{2u^{3/2}}\frac{du}{dx} + \sin x\, u^{-\frac{1}{2}} = u^{-\frac{3}{2}}\cos x$$

$$\boxed{\frac{du}{dx} - 2\sin x\, u = -2\cos x}$$

This is a linear DE

2-) $\frac{dy}{dx} - 2\sin x\, y = -2\sin x$

We multiply both sides by the integrating factor

$$e^{-\int 2\sin x dx} = e^{2\cos x}.$$

We find

$$\frac{d}{dx}\left(y e^{2\cos x}\right) = -2\sin x\, e^{2\cos x}$$

$$\Rightarrow y e^{2\cos x} = -\int 2\sin x\, e^{2\cos x} dx + C$$

$$= e^{2\cos x} + C$$

So that,

$$\boxed{y = 1 + Ce^{-2\cos x}, \quad x \in (-\infty, +\infty)}$$

Now, from $y(0) = 1$, we get $1 = 1 + Ce^{-2} \Rightarrow C = 0$

Thus, $\boxed{y = 1}$