MATH 202.13 (092)

Quiz 2 (Chap. 2.4-4.1.2) Duration: 20mn

Name: **ID number:**

1.) Find a suitable substitution that transforms the DE $(2x^2+3xy)dx+5y^2dy=0$ **into a separable DE. Find the new separable equation, but do not solve it.**

2.) Solve the DE $2xdy+(1+y^2)dx=0$ **subject to** $y(e)=0$.

Solution

Let $M(x,y)=2x^2+3xy$

and $N(x,y)=5y^2$

Both are homogeneous function of degree 2, so that this is an homogeneous DE.

We set $y=ux$.

We have $dy=x\,du+u\,dx$.

Substituting into the DE, we find

$$(2x^2+3x^2u)dx+5x^2u^2(x\,du+u\,dx)=0$$

$$x^2(2+3u+5u^3)dx+5u^2x^3du=0$$

$$\boxed{\frac{5u^2}{2+3u+5u^3}du+\frac{dx}{x}=0}$$

This is a separable DE.

2) We write the DE into the form

$$\frac{dy}{1+y^2}+\frac{dx}{2x}=0$$

$$\Rightarrow \int\frac{dy}{1+y^2}+\int\frac{dx}{2x}=0$$

$$\arctan y+\frac{1}{2}\ln|x|=C$$

So that

$$\boxed{\arctan y+\frac{1}{2}\ln x=C,\quad x>0}$$

Using now the condition $y(e)=0$, we find

$$\arctan 0+\frac{1}{2}\ln e=C$$

$$0+\frac{1}{2}=C \Rightarrow C=\frac{1}{2}$$

So, that the solution is

$$\boxed{\arctan y+\frac{1}{2}\ln x=\frac{1}{2},\quad x>0}$$