Important Note

Show all work.
Use of programmable calculator is not allowed.
Mobiles and paging devices should not be carried during examination.
Q1) (a) If \( \mathbf{r} = \langle x, y, z \rangle \) and \( \mathbf{a} = \langle a_1, a_2, a_3 \rangle \) is a constant vector, show that
\[
\nabla \times [(\mathbf{r} \cdot \mathbf{r}) \mathbf{a}] = 2(\mathbf{r} \times \mathbf{a}).
\]

Q1) (b) Find a vector giving direction in which \( f(x, y, z) = 2x^3 - xy + y^2 + z^3 \) increases most rapidly at \( P(1, -1, 2) \). Give the maximum rate.

*increases most rapidly*
Q2) Use Green’s theorem to evaluate \( \oint_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is boundary of the region enclosed by the circle \( x^2 + 2x + y^2 = 0 \), \( \mathbf{F} = (x + 2y)\mathbf{i} + xy\mathbf{j} \). Make a sketch of the curves showing its orientation. (4)
Q3) Verify the Stokes’ theorem in the case of the vector field $\mathbf{F} = xy \mathbf{k}$ and $S$ is the surface formed by $z = x^2 + y^2, 0 \leq z \leq 4$. Make a sketch of the surface showing its orientation.
Q4) Use the divergence theorem to evaluate the flux integral $\iint_S E \cdot n \, ds$ where the surface $S$ is portion of the sphere $z = \sqrt{9 - x^2 - y^2}$ lying inside the cylinder $x^2 + y^2 = 4$ above $xy$-plane and $E = e^{-z} \cos y \hat{i} + z^4 \hat{j} + \frac{1}{3}z^3 \hat{k}$.

(4)
Q5) Show that the integral \[ \int_{(0,0,0)}^{(1,1,1)} 2xz \, dx + 2yz \, dy + (x^2 + y^2) \, dz \] is independent of path. Then evaluate the integral.