No programmable calculators and mobile phones allowed in the examination hall. For all questions show calculations in support of your answers.
Q1

(a): Compute directional derivative of $\emptyset(x, y, z) = z \sin(xy)$ in the direction of the vector $\hat{i} + 2\hat{k}$ at the point $\left(\frac{\pi}{2}, 2, 1\right)$. 3 Points
(b): Evaluate $12 \int_C \left( \frac{x^2}{y} \right) ds$ with $C$ given by $x = y = t$ and $z = t^2$ for $1 \leq t \leq 2$.  4 Points
Q2. Use Green’s theorem to evaluate \( \oint_C \vec{F} \cdot d\vec{R} \), where \( \vec{F} = yx \hat{i} + x \hat{j} \) and \( C \) is the triangle with vertices (-1, 0), (1, 0) and (0, 2). The curve is oriented counter-clockwise.  

7 Points
Q3. Evaluate the surface integral $\int \int_{\Sigma} 2z \, d\sigma$, where $\Sigma$ is the part of the plane $2x + 2y + z = 8$ lying above the rectangle $1 \leq x \leq 1$ and $0 \leq y \leq 1$. 

6 Points
Q4. Use Gauss divergence theorem to evaluate \[ \iint_S \mathbf{F} \cdot \mathbf{N} \, d\sigma \], where \[ \mathbf{F} = \frac{x^2 \mathbf{i} + y \mathbf{j} + z^2 \mathbf{k}}{2} \] and \( S \) is the hemisphere of radius 3 about (-1, -1, -3).  

6 Points
Q5. Check if \( \vec{F} = (\cos x + 2y)\vec{i} + (\sin y + 2x)\vec{j} \) is conservative? If so, use independence of path and potential theory to evaluate \( \int_{C} \vec{F} \cdot d\vec{R} \) for \( C \) any closed path between \((\pi/2, \pi/2)\) to \((\pi, \pi)\).

7 Points
Q6. Use Stokes' theorem to compute \( \oint_C \vec{F} \cdot d\vec{R} \), where \( \vec{F} = xyz \mathbf{k} \) and \( \Sigma \) consists of the cone 

\[ z = \sqrt{x^2 + y^2} \text{ with } z \geq 0 \text{ and } x^2 + y^2 \leq 25. \] 

7 Points