1. Find the relative maxima and minima of the function \( f : \mathbb{R}^2 \to \mathbb{R} \) given by
\[
f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.
\]

2. Let \( f \) be the function given in problem 1. Find \( \nabla f \) and \( df(x, h) \) for \( h = (1, 1) \in \mathbb{R}^2 \).

3. Consider the sequence \( \{f_n\} \) of functions defined by \( f_n(x) = nxe^{-nx^2} \) on the interval \( I = [0, 1] \). Find the pointwise limit of the sequence and study whether the convergence is uniform or not.

4. Find the Fourier series for \( f(x) = x + x^2 \) on \( I = [-\pi, \pi] \) and then show that
\[
\sum_{n=1}^{+\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.
\]

5. Let \( f(x, y) = \frac{\sin xy}{x(x^2 + 1)} \) for \( x > 0, y > 0 \). Define \( \phi(y) = \int_{0}^{+\infty} f(x, y)dx \). Show that \( \phi \) is well defined and satisfies
\[
\phi'' - \phi = -\frac{\pi}{2}, \quad \phi(0) = 0, \quad \phi'(0) = \frac{\pi}{2}.
\]
Show that \( \phi(y) = \frac{\pi}{2} (1 - e^{-y}) \) (hint: \( \int_{0}^{+\infty} \frac{\sin \omega}{\omega}d\omega = \frac{\pi}{2} \)).