1. Show that the sequence of functions \( f_n \), defined by \( f_n(x) = \frac{x}{1 + n^2 x^2} \), converges uniformly to a function \( f \). Is it true that \( f(x) = \lim_{n \to \infty} f_n(x) \)?

2. Show that \( f_n(x) = (n+2)(n+1)x^n(1-x) \) converges to \( f(x) = 0 \). Find \( \lim_{n \to \infty} \int_0^1 f_n(x) \, dx \). Justify your answer.

3. Assume that \( \sum_{n=1}^{\infty} |b_n| \) is convergent. Prove that both series \( \sum_{n=1}^{\infty} b_n \sin nx \) and \( \sum_{n=1}^{\infty} nb_n \cos nx \) are uniformly convergent.