

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**Math 430 Major Exam 2**  
**First Semester 2009–2010(092)**  
**March 31, 2010**

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Name: \_\_\_\_\_

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1. The function  $\sinh z$  is defined by

$$\sinh z = \frac{e^z - e^{-z}}{2}.$$

Find a formula for  $\sinh^{-1}(z)$  in terms of the logarithm function.

2. Let  $p(z) = (z - z_1)^{d_1} (z - z_2)^{d_2} \cdots (z - z_r)^{d_r}$ . So  $\frac{p'(z)}{p(z)} = \frac{d_1}{(z - z_1)} + \cdots + \frac{d_r}{(z - z_r)}$ .

If each  $d_i$  is real and positive and  $\text{Im}(z_k) > 0$ ,  $k = 1, \dots, r$ , show that  $R(z)$  has no zeros in the lower half plane by showing that  $\text{Im } R(z) > 0$  for  $\text{Im } z < 0$ .

3. (a) Find all solutions of the equation  $\sin z = \cos z$ .

(b) Find  $\int_{\Gamma} \frac{3z - 2}{(z^2 - z)} dz$ , where  $\Gamma$  is a simple closed contour containing 0 and 1 in its interior.

4. Let  $I = \oint_{|z|=2} \frac{dz}{z^2(z-1)^3}$ . Show

(a) for every  $R > 2$ ,  $I = I(R)$  where  $I(R) = \oint_{|z|=R} \frac{dz}{z^2(z-1)^3}$ .

(b)  $|I(R)| \leq \frac{2\pi}{R(R-1)^3}$  for  $R > 2$

(c)  $\lim_{R \rightarrow \infty^+} I(R) = 0$

(d)  $I = 0$ .