Prob. 1
Find the Fourier series of
\[ f(t) = \sinh \left[ a \left( \frac{\pi}{2} - |t| \right) \right], \quad -\pi < t < \pi. \]

Prob. 2
By taking the appropriate closed contour, find the inverse Fourier transform of \( \frac{1}{w^2 - 3iw - 3} \).

Prob. 3
Solve \( f(t) = 6t + 4 + \int_0^t f(x)(x - t)^2 dx \).

Prob. 4
Consider
\[
\begin{cases}
  y'' + (\lambda - a^2)y = 0, & 0 < x < 1 \\
  y'(0) + ay(0) = 0, & y'(1) + ay(1) = 0
\end{cases}
\]

a) Show that this is a regular S-L problem.

b) Show that \( \lambda_0 = 0 \) implies \( y_0(x) = e^{-ax} \) and \( \lambda_n = a^2 + n^2\pi^2 \) implies \( y_n(x) = a \sin(n\pi x) - n\pi \cos(n\pi x), \quad n = 1, 2, \ldots \).
c) Given a function $f$, show that we can expand it as follows

$$f(x) = C_0 e^{-ax} + \sum_{n=1}^{\infty} C_n [a_n \sin(n\pi x) - n\pi \cos(n\pi x)]$$

where

$$C_n = (1 - e^{-2a})C_0 = 2a \int_0^1 f(x) e^{-ax} \, dx$$

and

$$(a^2 + n^2 \pi^2)C_n = 2 \int_0^1 f(x) [a \sin(n\pi x) - n\pi \cos(n\pi x)] \, dx.$$

**Prob 5**

From the recurrence formulas show that

$$J_0''(x) = \frac{J_0(x)}{x} + \left( \frac{2}{x^2} - 1 \right) J_0'(x).$$