1. Let $L$ and $M$ be given by

$$Lu(x) = -(p(x)u'(x))' + q(x)u(x),$$

and

$$M = \{u \in C^2([0, 1]) : u(0) = u(1), u'(0) = u'(1)\}.$$

Find $L^*$ and $M^*$.

2. Let

$$Lu(x) = (e^x u'(x))', \quad B_1 u = u(0), \ B_2 u = u'(0).$$

Show that

$$(e^x y'(x))' z - (e^x z'(x))' y = [e^x (zy' - z'y)'].$$

Find $L^*, B_1^*, B_2^*$. Find the Green’s function $g(x, \tau)$ for $Lu(x) = 0, B_1 u = B_2 u = 0$.

3. The next problems are from the Book by I. Stakgold. Do problems: 1.2 page 104 + 1.5 and 1.6 page 105 + 2.4 page 118 + 2.5 page 119.