

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102

Exam I

093

Monday 19/07/2010

Net Time Allowed: 120 minutes

MASTER VERSION

1. To estimate the area under the graph of $f(x) = x \sin x$ from $x = 0$ to $x = \pi$ using four rectangles and the right endpoints we get

(a) $\frac{\pi^2(1 + \sqrt{2})}{8}$

(b) $\frac{\pi^2(2 - \sqrt{2})}{4}$

(c) $\frac{\pi^2(\sqrt{2} - 1)}{8}$

(d) $\frac{\pi^2(2 - \sqrt{2})}{8}$

(e) $\frac{\sqrt{2}\pi^2}{8}$

2. $\int \frac{\sqrt{x} + \sqrt[4]{x}}{\sqrt[3]{x}} dx =$

(a) $\frac{6}{7}x^{7/6} + \frac{12}{11}x^{11/12} + c$

(b) $-\frac{1}{x} + \frac{1}{2}x^2 + c$

(c) $42x^{7/6} + 132x^{11/12} + c$

(d) $\ln|x| + \frac{1}{2}x^2 + c$

(e) $\frac{6}{5}x^{5/6} + \frac{12}{19}x^{19/12} + c$

3. $\int_0^1 \frac{3x^3 + x^2 - 18x - 6}{3x + 1} dx =$

(a) $-\frac{17}{3}$

(b) $-\frac{19}{3}$

(c) $\frac{14}{3}$

(d) $-\frac{11}{3}$

(e) $\frac{5}{3}$

4. $\int (\tan x + \cot x) dx =$

(a) $\ln |\tan x| + c$

(b) $\ln |\cot x| + c$

(c) $\ln |\sec x| + c$

(d) $\ln |\csc x| + c$

(e) $\ln |\sec x + \csc x| + c$

5. If $v(t) = (t - 3)$ m/s is the velocity of a particle moving on a line at time t in seconds, then the total distance traveled by the particle during the time interval $[0, 4]$ is

- (a) $5m$
- (b) $9m$
- (c) $6m$
- (d) $3m$
- (e) $14m$

6. $\int \frac{\sin^{-1}\left(\frac{x}{2}\right)}{\sqrt{4-x^2}} dx =$

- (a) $\frac{1}{2} \left(\sin^{-1}\left(\frac{x}{2}\right) \right)^2 + c$
- (b) $\ln \left| \sin^{-1}\left(\frac{x}{2}\right) \right| + c$
- (c) $4 \left(\sin^{-1}\left(\frac{x}{2}\right) \right)^2 + c$
- (d) $\frac{1}{4-x^2} + c$
- (e) $\sqrt{4-x^2} \sin^{-1}\left(\frac{x}{2}\right) + c$

7. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(3 - \frac{2i}{n} \right)^2 =$

(a) $\frac{13}{3}$

(b) $\frac{22}{3}$

(c) $\frac{14}{3}$

(d) $\frac{52}{3}$

(e) 0

8. The area of the region enclosed by the curves $y = \sqrt{x}$, $y = x$, $x = 0$, and $x = 4$ is equal to

(a) 3

(b) $\frac{3}{2}$

(c) 5

(d) $\frac{5}{2}$

(e) $\frac{10}{3}$

9. If $\int_4^7 f(x)dx = 5$, then $\int_1^4 \frac{f(3\sqrt{x} + 1)}{\sqrt{x}} dx =$

(a) $\frac{10}{3}$

(b) 30

(c) $\frac{35}{3}$

(d) 20

(e) $\frac{25}{3}$

10. The area of the region enclosed by $2x + y^2 = 3$ and $x - y = 0$ is

(a) $\left[\frac{3}{2}y - \frac{1}{2}y^2 - \frac{1}{6}y^3\right]_{-3}^1$

(b) $\left[\frac{1}{2}y - \frac{3}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^3$

(c) $\left[\frac{3}{2}y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-3}^1$

(d) $\left[\frac{3}{2}y - \frac{1}{3}y^2 - \frac{1}{3}y^3\right]_{-3}^1$

(e) $\left[\frac{3}{2}y - \frac{2}{3}y^2 - \frac{1}{6}y^3\right]_{-1}^3$

11. The base of a solid is the region bounded by the parabola $y = 1 - x^2$ and the x -axis. Each cross section perpendicular to the x -axis is a square. The volume of the solid is

(a) $\frac{16}{15}$

(b) $\frac{4}{3}$

(c) $\frac{2}{3}$

(d) $\frac{15}{12}$

(e) $\frac{8}{15}$

12. The volume of the solid obtained by rotating the region bounded by the graphs of $y = \tan x$, $x = \frac{\pi}{4}$ and $y = 0$ about the x -axis is equal to

(a) $\frac{4\pi - \pi^2}{4}$

(b) π

(c) $2\pi^2 - 4\pi$

(d) $\frac{3\pi^2 - 4\pi}{4}$

(e) $\frac{\pi}{4}$

13. $\int_{-1}^0 (x+1) e^{-x(x+2)} dx =$

(a) $\frac{1}{2}(e-1)$

(b) $\frac{3}{2}e$

(c) $\frac{1}{4}(e+1)$

(d) $\frac{1}{4}(e-1)$

(e) $1-e$

14. An expression for the area under the graph of $f(x) = 3x^2 + 5x$, $0 \leq x \leq 2$, as a limit and using right endpoints is

(a) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{24i^2}{n^3} + \frac{20i}{n^2} \right]$

(b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{6i^2}{n^3} + \frac{10i}{n^2} \right]$

(c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{3i^2}{n^3} + \frac{20i}{n^2} \right]$

(d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{24i^2}{n^3} + \frac{5i}{n^2} \right]$

(e) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{3i^2}{4n^3} + \frac{5i}{2n^2} \right]$

15. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\csc x)(3 \sin 2x + 5 \sin x) dx =$

(a) $3 + \frac{5\pi}{3}$

(b) $1 + \frac{\pi}{3}$

(c) $3 + \frac{2\pi}{3}$

(d) $1 + \frac{5\pi}{3}$

(e) 5π

16. If the plane region enclosed by the graphs of $y = x$ and $y = x^2$ is revolved about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^1 \pi(2\sqrt{y} - y - y^2) dy$

(b) $\int_{-1}^1 \pi(y - y^2) dy$

(c) $\int_0^1 \pi(4\sqrt{y} + 6y - y^2) dy$

(d) $\int_{-1}^1 \pi(y - y^2 - 2) dy$

(e) $\int_0^1 \pi(2\sqrt{y} - 6y - y^2) dy$

17. The equation of the tangent line to the graph of

$$f(x) = \int_{\sqrt{x}}^{x^3} e^{u^2} du \text{ at } x = 1 \text{ is}$$

(a) $y = \frac{5e}{2}x - \frac{5e}{2}$

(b) $y = \frac{3e}{2}x - \frac{3e}{2}$

(c) $y = \frac{2e}{3}x - \frac{2e}{3}$

(d) $y = \frac{5e}{2}x - \frac{e}{2}$

(e) $y = \frac{5e}{2}x + \frac{e}{2}$

18. If $k = \int_0^{\frac{\pi}{2}} e^{-\sin x} dx$, then

(a) $\frac{\pi}{2e} \leq k \leq \frac{\pi}{2}$

(b) $0 \leq k \leq \frac{\pi}{2e}$

(c) $k \geq \frac{\pi}{2}$

(d) $k \geq \frac{\pi}{2e} + \frac{\pi}{2}$

(e) $-\frac{\pi}{2e} \leq k \leq \frac{\pi}{2e}$

19. The value of $\int_{-\pi}^{\pi} (4 + 3 \sin x) \sqrt{\pi^2 - x^2} dx$ is equal to :
(Hint : write the integral as a sum of two integrals and interpreting one of these integrals in terms of an area)

(a) $2\pi^3$

(b) 0

(c) $\frac{\pi^3}{2}$

(d) $8\pi^3$

(e) $\frac{\pi^3}{4}$

20. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{1}{n} + \frac{i}{n^2} + \frac{1}{n} e^{1+\frac{i}{n}} \right] =$

(a) $\int_1^2 (x + e^x) dx$

(b) $\int_1^2 (1 + x + e^x) dx$

(c) $\int_0^1 (x + e^x) dx$

(d) $\int_0^1 (1 + x + e^x) dx$

(e) $\int_1^2 (x + x^2 e^x) dx$