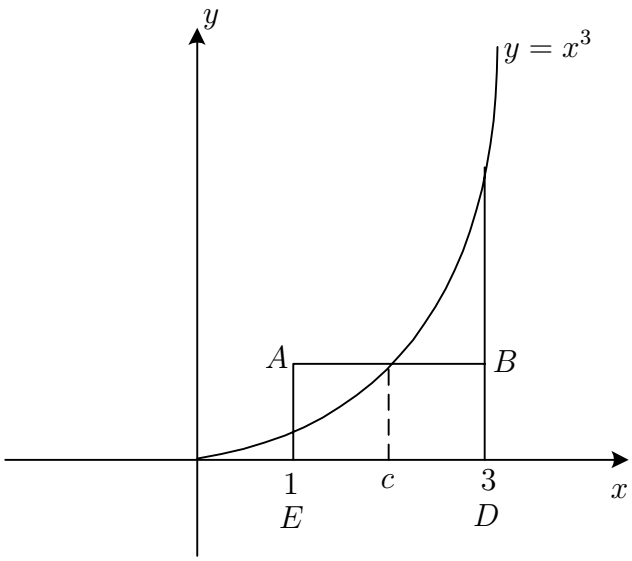
1. (5-points) Find a number c, $1 < c < 3$, such that the area of the rectangle $ABDE$ in the adjacent figure is equal to the area of the region bounded above by $y = x^3$, and bounded below by $y = 0$ from $x = 1$ to $x = 3$.



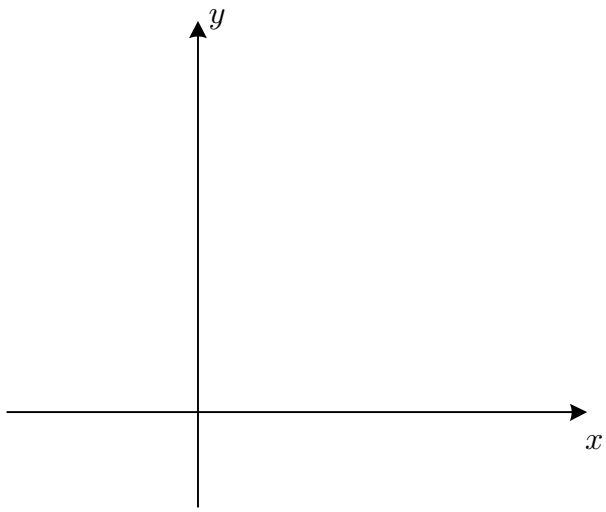
2. (5-points) List the first five terms of the sequence $\{a_n\}$ where $a_1 = 1$ and $a_{n+1} = \dfrac{1}{1 + a_n}$.

3. (5-ponts) Find a formula for the general term a_n of the sequence $\left\{\dfrac{2^2}{2 \cdot 3}, \dfrac{4^2}{3 \cdot 5}, \dfrac{6^2}{4 \cdot 7}, \dfrac{8^2}{5 \cdot 9}, \ldots\right\}$. Then determine whether the sequence converges or diverges.

4. (12-points) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the graphs of

   $y = e^{-x},\ x = 0,\quad y = 0,\quad$ and $\quad x = 1\quad$ about the line $\quad x = -2.$

   [Sketch the region and a typical rectangle]



5. (4-points) Write out the form of the partial fraction decomposition of $\dfrac{x+5}{x^2(x^2-x-2)(x^2+5)^2}$. Do not determine the numerical values of the coefficients.

6. Evaluate the following integrals:

(a) (6-points) $\displaystyle\int \frac{\sqrt{\csc\theta}}{\sec^3\theta}\,d\theta.$

(b) (6-points) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx.$

(c) (8-points) $\int x \tan^{-1}\left(\frac{1}{x}\right)\, dx.$

(d) (8-points) $\int \frac{x^2}{(4-9x^2)^{3/2}}\, dx.$

(e) (12-points) $\displaystyle\int \frac{7x^2 + x}{(3x - 1)(x^2 + 1)}\, dx.$

7. (8-points) Show that the integral test is applicable to the series $\sum_{n=0}^{\infty} \frac{1}{n^2+4n+5}$, then use it to determine whether the series converges or diverges.

8. (5-points) Evaluate $\int_0^4 \frac{1}{(x-1)^2}\,dx$ if possible.

9. (6-points) The n-th partial sum S_n of the seris $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \dfrac{n-1}{n+1}$. Find a_n and the sum of the series.

10. Determine whether each of the following series is convergent or divergent. Find the sum of the series if it is convergent.

(a) (5-points) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4\sqrt{n^3} + 5\sqrt[3]{n}}{n^2} \right).$

(b) (5-points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n-2}}{5^{n-1}}.$