King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
093
Monday 19/07/2010
Net Time Allowed: 120 minutes

MASTER VERSION
1. To estimate the area under the graph of $f(x) = x \sin x$ from $x = 0$ to $x = \pi$ using four rectangles and the right endpoints we get

(a) $\frac{\pi^2(1 + \sqrt{2})}{8}$

(b) $\frac{\pi^2(2 - \sqrt{2})}{4}$

(c) $\frac{\pi^2(\sqrt{2} - 1)}{8}$

(d) $\frac{\pi^2(2 - \sqrt{2})}{8}$

(e) $\frac{\sqrt{2}\pi^2}{8}$

2. $\int \frac{\sqrt{x} + \sqrt{x}}{\sqrt[3]{x}} \, dx =$

(a) $\frac{6}{7}x^{7/6} + \frac{12}{11}x^{11/12} + c$

(b) $-\frac{1}{x} + \frac{1}{2}x^2 + c$

(c) $42x^{7/6} + 132x^{11/12} + c$

(d) $\ln |x| + \frac{1}{2}x^2 + c$

(e) $\frac{6}{5}x^{5/6} + \frac{12}{19}x^{19/12} + c$
3. \[ \int_0^1 \frac{3x^3 + x^2 - 18x - 6}{3x + 1} \, dx = \]

(a) \( -\frac{17}{3} \)

(b) \( -\frac{19}{3} \)

(c) \( \frac{14}{3} \)

(d) \( -\frac{11}{3} \)

(e) \( \frac{5}{3} \)

4. \[ \int (\tan x + \cot x) \, dx = \]

(a) \( \ln |\tan x| + c \)

(b) \( \ln |\cot x| + c \)

(c) \( \ln |\sec x| + c \)

(d) \( \ln |\csc x| + c \)

(e) \( \ln |\sec x + \csc x| + c \)
5. If \( v(t) = (t - 3) \) m/s is the velocity of a particle moving on a line at time \( t \) in seconds, then the total distance traveled by the particle during the time interval \([0, 4]\) is

(a) \( 5m \)

(b) \( 9m \)

(c) \( 6m \)

(d) \( 3m \)

(e) \( 14m \)

6. \( \int \frac{\sin^{-1}\left(\frac{x}{2}\right)}{\sqrt{4 - x^2}} \, dx = \)

(a) \( \frac{1}{2} \left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 + c \)

(b) \( \ln \left| \sin^{-1}\left(\frac{x}{2}\right) \right| + c \)

(c) \( 4 \left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 + c \)

(d) \( \frac{1}{4 - x^2} + c \)

(e) \( \sqrt{4 - x^2} \sin^{-1}\left(\frac{x}{2}\right) + c \)
7. \[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \left( 3 - \frac{2i}{n} \right)^2 =
\]

(a) \(\frac{13}{3}\)

(b) \(\frac{22}{3}\)

(c) \(\frac{14}{3}\)

(d) \(\frac{52}{3}\)

(e) 0

8. The area of the region enclosed by the curves \(y = \sqrt{x}\), \(y = x, x = 0, \text{ and } x = 4\) is equal to

(a) 3

(b) \(\frac{3}{2}\)

(c) 5

(d) \(\frac{5}{2}\)

(e) \(\frac{10}{3}\)
9. If \( \int_{4}^{7} f(x) \, dx = 5 \), then \( \int_{1}^{4} \frac{f(3\sqrt{x} + 1)}{\sqrt{x}} \, dx = \)

(a) \( \frac{10}{3} \)

(b) 30

(c) \( \frac{35}{3} \)

(d) 20

(e) \( \frac{25}{3} \)

10. The area of the region enclosed by \( 2x + y^2 = 3 \) and \( x - y = 0 \) is

(a) \( \left[ \frac{3}{2}y - \frac{1}{2}y^2 - \frac{1}{6}y^3 \right]_{-3}^{1} \)

(b) \( \left[ \frac{1}{3}y - \frac{3}{2}y^2 - \frac{5}{3}y^3 \right]_{3}^{1} \)

(c) \( \left[ \frac{3}{2}y + \frac{1}{2}y^2 - \frac{1}{3}y^3 \right]_{-3}^{1} \)

(d) \( \left[ \frac{3}{2}y - \frac{1}{3}y^2 - \frac{1}{3}y^3 \right]_{-3}^{1} \)

(e) \( \left[ \frac{3}{2}y - \frac{2}{3}y^2 - \frac{1}{6}y^3 \right]_{-1}^{3} \)
11. The base of a solid is the region bounded by the parabola $y = 1 - x^2$ and the $x-$ axis. Each cross section perpendicular to the $x-$ axis is a square. The volume of the solid is

(a) $\frac{16}{15}$

(b) $\frac{4}{3}$

(c) $\frac{2}{3}$

(d) $\frac{15}{12}$

(e) $\frac{8}{15}$

12. The volume of the solid obtained by rotating the region bounded by the graphs of $y = \tan x, x = \frac{\pi}{4}$ and $y = 0$ about the $x-$ axis is equal to

(a) $\frac{4\pi - \pi^2}{4}$

(b) $\pi$

(c) $2\pi^2 - 4\pi$

(d) $\frac{3\pi^2 - 4\pi}{4}$

(e) $\frac{\pi}{4}$
13. \[ \int_{-1}^{0} (x + 1) \, e^{-x(x+2)} \, dx = \]

(a) \( \frac{1}{2} (e - 1) \)

(b) \( \frac{3}{2} e \)

(c) \( \frac{1}{4} (e + 1) \)

(d) \( \frac{1}{4} (e - 1) \)

(e) \( 1 - e \)

14. An expression for the area under the graph of \( f(x) = 3x^2 + 5x \), \( 0 \leq x \leq 2 \), as a limit and using right endpoints is

(a) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{24i^2}{n^3} + \frac{20i}{n^2} \right] \)

(b) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{6i^2}{n^3} + \frac{10i}{n^2} \right] \)

(c) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{3i^2}{n^3} + \frac{20i}{n^2} \right] \)

(d) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{24i^2}{n^3} + \frac{5i}{n^2} \right] \)

(e) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{3i^2}{4n^3} + \frac{5i}{2n^2} \right] \)
15. \[ \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\csc x)(3 \sin 2x + 5 \sin x)dx = \]

(a) \[3 + \frac{5\pi}{3}\]

(b) \[1 + \frac{\pi}{3}\]

(c) \[3 + \frac{2\pi}{3}\]

(d) \[1 + \frac{5\pi}{3}\]

(e) \[5\pi\]

16. If the plane region enclosed by the graphs of \( y = x \) and \( y = x^2 \) is revolved about the line \( x = -1 \), then the volume of the solid generated is given by

(a) \[\int_{0}^{1} \pi(2\sqrt{y} - y - y^2)dy\]

(b) \[\int_{-1}^{1} \pi(y - y^2)dy\]

(c) \[\int_{0}^{1} \pi(4\sqrt{y} + 6y - y^2)dy\]

(d) \[\int_{-1}^{1} \pi(y - y^2 - 2)dy\]

(e) \[\int_{0}^{1} \pi(2\sqrt{y} - 6y - y^2)dy\]
17. The equation of the tangent line to the graph of 
\[ f(x) = \int_{\sqrt{x}}^{x^2} e^{u} \, du \] at \( x = 1 \) is

(a) \( y = \frac{5e}{2} x - \frac{5e}{2} \)
(b) \( y = \frac{3e}{2} x - \frac{3e}{2} \)
(c) \( y = \frac{2e}{3} x - \frac{2e}{3} \)
(d) \( y = \frac{5e}{2} x - \frac{e}{2} \)
(e) \( y = \frac{5e}{2} x + \frac{e}{2} \)

18. If \( k = \int_{0}^{\frac{\pi}{2}} e^{-\sin x} \, dx \), then

(a) \( \frac{\pi}{2e} \leq k \leq \frac{\pi}{2} \)
(b) \( 0 \leq k \leq \frac{\pi}{2e} \)
(c) \( k \geq \frac{\pi}{2} \)
(d) \( k \geq \frac{\pi}{2e} + \frac{\pi}{2} \)
(e) \( -\frac{\pi}{2e} \leq k \leq \frac{\pi}{2e} \)
19. The value of \( \int_{-\pi}^{\pi} (4 + 3\sin x)\sqrt{\pi^2 - x^2} \, dx \) is equal to:
(Hint: write the integral as a sum of two integrals and interpreting one of these integrals in terms of an area)

(a) \( 2\pi^3 \)
(b) \( 0 \)
(c) \( \frac{\pi^3}{2} \)
(d) \( 8\pi^3 \)
(e) \( \frac{\pi^3}{4} \)

20. \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{1}{n} + \frac{i}{n^2} + \frac{1}{n} e^{1+\frac{i}{n}} \right] = \)

(a) \( \int_{1}^{2} (x + e^x) \, dx \)
(b) \( \int_{1}^{2} (1 + x + e^x) \, dx \)
(c) \( \int_{0}^{1} (x + e^x) \, dx \)
(d) \( \int_{0}^{1} (1 + x + e^x) \, dx \)
(e) \( \int_{1}^{2} (x + x^2e^x) \, dx \)