King Fahd University of Petroleum and Minerals
FINAL
MATH. 202-093
(3 Hours)

Name:
ID:

**Prob. 1** (11 points)
Solve the nonhomogeneous system

\[ X' = \begin{pmatrix} -3 & 1 \\ 2 & -2 \end{pmatrix} X + \begin{pmatrix} 2t \\ e^t \end{pmatrix} \]

on \((-\infty, \infty)\).
Prob. 2 (11 points)
Find two linearly independent power series solutions of the differential equation
\[ y'' - 2xy = 0 \]
about the ordinary point \( x = 0 \).
Prob. 3 (11 points)
Find one power series about zero of the equation

\[ x(1 - x)y'' - 3xy' - y = 0. \]

You should be able to sum this power series to write down the solution explicitly. Now use the reduction of order method to find a second solution.

**Hint:**

\[
\frac{d}{dx} \left( \frac{1}{1 - x} \right) = \frac{d}{dx} \left( 1 + x + x^2 + x^3 + x^4 + ... \right) \\
= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + ...
\]
Prob. 4 (11 points)
(a) Use the series expansion (definition of $e^{At}$) to compute $e^{At}$ when
\[ A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

(b) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$. Show that $A^3 = 0$ then obtain $e^{At}$.

(c) Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 2 & -1 \end{pmatrix}$. Compute $e^{At}$.
Prob. 5 (11 points)
Solve the differential equation

\[ y' = y \tan t + \sec t \]
Prob. 6 (11 points)
Solve the differential equation
\[ t^2 y' = yt + y \sqrt{t^2 + y^2} \]

**Hint:** \[ \int \frac{dv}{v \sqrt{1 + v^2}} = \ln \left| \frac{v}{1 + \sqrt{1 + v^2}} \right| \]
Prob. 7 (11 points)
Consider the differential equation

\[(1 + x^2)y'' - 2xy' + 2y = 0\]

(1) Show that \(y_1(x) = x\) is solution of the differential equation
(2) Find a second solution \(y_2\) independent of \(y_1\)
(3) Obtain the general solution of

\[(1 + x^2)y'' - 2xy' + 2y = (1 + x^2)^2\]
Prob. 8 (11 points)
Solve the system of equations
\[
\begin{align*}
u'(t) &= -2u(t) + 2v(t) \\
v'(t) &= u(t) - v(t)
\end{align*}
\]