King Fahd University of Petroleum & Minerals  
Department of Mathematics & Statistics  
Math 301  Major Exam 2  
The Summer Semester of 2009-2010 (093)  
Time Allowed: 120 Minutes

Name: ______________________  ID#: ______________________

Instructor: Muhammad Yousuf  Serial #: ______________________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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<th>Question #</th>
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Q:1 (a) (5 points) Use Laplace transform to solve the initial value problem

\[ y'' - 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 3. \]

(b) (5 points) Find Laplace transform of \( f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \frac{\pi}{2} \\ \sin t & \text{if } t \geq \frac{\pi}{2} \end{cases} \).
Q:2 (a) (5 points) Find Laplace transform $\mathcal{L}\{te^{-3t} \cos^2 t\}$.

(b) (5 points) Find Laplace transform $\mathcal{L}^{-1} \left\{ \frac{1}{s^2 (s - 2)} \right\}$.

(c) (6 points) Use Laplace transform to solve the initial value problem

$$y'' + y = \delta (t - 2\pi) + \delta (t - 3\pi), \quad y(0) = 1, \quad y'(0) = 0.$$
Q:3 (10 points) Show that the set of functions \( \left\{ 1, \cos \left( \frac{n\pi}{p} \right) x \right\} \) is an orthogonal set on \([0, p]\) for \( n = 1, 2, 3, \cdots \). Also find norm of each function. (Justify your answer with reason).
Q:4 (12 points) Find Fourier series expansion of \( f(x) = \begin{cases} 1 & \text{if } -1 < x < 0 \\ x & \text{if } 0 \leq x < 1 \end{cases} \).
Q:5 (a) (10 points) Find eigenvalues and eigenfunctions of the boundary value problem
\[ x^2 y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(5) = 0. \]

(b) (3 points) Write the given differential equation into self-adjoint form.

(c) (2 points) Write the orthogonality condition.
Q:6 (12 points) Solve the Heat equation using separation constant $\lambda = \alpha^2$, $\alpha > 0$

\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \]

subject to the conditions $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = \begin{cases} 
1 & \text{if } 0 < x < \frac{L}{2} \\
0 & \text{if } \frac{L}{2} < x < L
\end{cases}$.